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Effect of Rotation on a Layer of Micro-Polar Ferromagnetic Dusty Fluid Heated from Below Saturating a Porous Medium

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ABSTRACT
This paper deals with the theoretical investigation of effect of rotation on micro-polar ferromagnetic dusty fluid layer heated from below in a porous medium. Linear stability analysis and normal mode analysis methods are used to find an exact solution for a flat micro-polar ferromagnetic fluid layer contained between two free boundaries . In case of stationary convection, the effect of various parameters like medium permeability parameter, non-buoyancy magnetization parameter, micro-polar coupling parameter, spin-diffusion parameter, micro-polar heat conduction parameter, dust particles parameter and rotation parameter has been analyzed and results are depicted graphically. In the absence of dust particles, rotation, micro-viscous effect and micro-inertia, the sufficient condition is obtained for non-oscillatory modes

I. INTRODUCTION

Magnetic fluids or Ferro-fluids are colloidal liquids made of neno-scale ferromagnetic, or ferri-magnetic particles suspended in a carrier fluid (usually an organic solvent or water). Each tiny particle is thoroughly coated with a surfactant to inhibit clumping. Large ferromagnetic particles can be ripped out of the homogeneous colloidal mixture, forming a separate clump of magnetic dust when exposed to strong magnetic fields. The magnetic attraction of nano-particles is weak enough that the surfactants Vander Waals force is sufficient to prevent magnetic clumping or agglomeration. Ferro-fluids usually do not retain magnetization in the absence of an externally applied field and thus are often classified as superparamagnets rather than ferromagnets.

Experimental and theoretical physicists and engineers gave significant contributions to ferrohydrodynamics and its application. An authoritative introduction to this subject has been discussed in detail in the celebrated monograph by **Rosensweig[8]**. Many authors **[2, 6, 7, 11, 19]** discussed the thermal convection in ferromagnetic fluids. **Scanlon** and **Segel[4]** have considered the effects of suspended particles on the onset of Bénard convection, whereas **Sunil** *et al.***[20]** have studied the effect of dust particles on thermal convection in ferromagnetic fluid saturating a porous medium by assuming isotropic properties. During the last half century, research on magnetic liquids has been very productive in many areas (earthquake protection, air bags, sealing of rotating shafts etc.) and the mechanism of controlling convection in a ferromagnetic fluids is important in material processing in space because of its applications to the possibility of producing various materials. Now-a-days micropolar ferromagnetic fluid stabilities have become an important field of research **[1, 5]**. Thermal convection in porous medium is also of great interest in chemical engineering, metallurgy and geophysics, electrochemistry and biomechanics**[3]. Sunil** *et al.***[15, 16, 18, 22, 23]** have discussed the marginal stability of micropolar ferromagnetic fluid saturating a porous medium and thermal convection in micropolar ferrofluid in the presence of rotation saturating a porous and non-porous medium. In the study of micropolar ferromagnetic fluid layers done by **Sunil** *et. al***[15, 16, 18, 22, 23]**, the fluid has been assumed to be clean (free from dust particles). However, in many geophysical situations, the fluid is often not pure but contains suspended/dust particles. The effect of dust particles on stability problems of micropolar ferromagnetic fluid through porous medium reflects its usefulness is several geophysical situations, chemical engineering, biomechanics and industry.

The effect of dust particles on ferromagnetic fluid for porous and non-porous medium has been studied by several anothers **[12-14, 17, 20, 21]**. **Reena** and **Rana[10]** have studied the effect of dust particles on rotating micropolar fluids heated from below saturating a porous medium. **Reena** and **Rana[9]** have studied the effect of dust particles on a layer of mircopolar ferromagnetic fluid heated from below saturating a porous medium.

Keeping in mind the usefulness of micropolar ferromagnetic fluids and their various applications in several fields given above, I have made an attempt to examine the effect of rotation on a layer of micropolar ferromagnetic dusty fluid heated from below saturating a porous medium and to the best my knowledge this problem is uninvestigated so far.

II. MATHEMATICAL FORMULATION

Consider an infinite, horizontal, electrically non-conducting incompressible thin micro-polar ferromagnetic fluid layer of thickness *d*, embeded in dust particles, heated from below as shown in figure below.

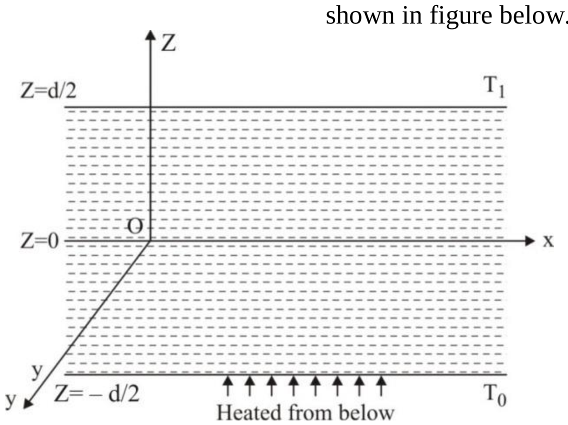

Fig. 1

The physical structure is one of infinite extent in the *x* and *y* directions bounded by the planes $z = -\frac{d}{2}$ and $z = \frac{d}{2}$. The upper boundary is held at fixed temperature $T = T_1$ and the lower boundary is held at constant temperature $T = T_0$ such that a steady adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. A strong but uniform magnetic field $\vec{\mathbf{H}} = (0, 0, H_0^{ext})$ is applied along *z*-direction. The whole system is acted upon by a gravity $\vec{\mathbf{g}} = (0,\ 0,\ -g)$ and is assumed to be rotating with uniform angular velocity $\vec{\mathbf{\Omega}} = (0,\ 0,\ \Omega_0)$ about *z*-axis.

This micro-polar ferromagnetic fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity $\in$ and the medium permeability κ_1, where the porosity is defined as the fraction of the total volume of the medium that is occupied by void space. Thus $1-\in$ is the fraction that is occupied by solid. For an isotropic medium the surface porosity (the fraction of void area to total area of a typical cross section) will normally be equal to $\in$. Here both the boundaries are taken to be free and perfect conductor of heat.

Within Boussinesq approximation, the mathematical equations governing the motion of a micro-polar ferromagnetic fluid saturating a porous medium following Darcy's law for the above model are as follows :

The equation of continuity for an incompressible micro-polar ferromagnetic fluid is

$$\nabla . \vec{\mathbf{q}} = 0 \qquad ...(1)$$

The equation of momentum for generalized Darcy model including the inertial forces is given by

$$\frac{\rho_0}{\in}\left[\frac{\partial \vec{\mathbf{q}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}} . \overline{\nabla})\vec{\mathbf{q}}\right] = -\nabla p - \rho g \,\hat{\mathbf{e}}_z - \frac{(\mu + \zeta)}{\kappa_1}\vec{\mathbf{q}} + \frac{KN}{\in}(\vec{\mathbf{q}}_d - \vec{\mathbf{q}}) + \nabla(\vec{\mathbf{H}} . \vec{\mathbf{B}}) + \frac{2\rho_0}{\in}(\vec{\mathbf{q}} \times \vec{\mathbf{\Omega}}) + \zeta(\nabla \times \vec{\mathbf{N}}) \qquad ...(2)$$

The equation of internal angular momentum is given by

$$\rho_0\, j\left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}} . \nabla)\vec{\mathbf{N}}\right] = (\alpha' + \beta')\nabla(\nabla . \vec{\mathbf{N}}) + \gamma' \nabla^2 \vec{\mathbf{N}} + \zeta\left(\frac{1}{\in}\overline{\nabla} \times \vec{\mathbf{q}} - 2\vec{\mathbf{N}}\right) + \mu_0(\vec{\mathbf{M}} \times \vec{\mathbf{H}}) \qquad ...(3)$$

The equation of energy is given by

$$\in\left[\rho_0 C_{V,H} - \mu_0 \vec{\mathbf{H}} . \left(\frac{\partial \vec{\mathbf{N}}}{\partial T}\right)_{V,H}\right]\left[\frac{\partial T}{\partial t} + (\vec{\mathbf{q}} . \nabla)T\right] + (1-\in)\rho_s C_s \frac{\partial T}{\partial t}$$

$$+\mu_0 T \left(\frac{\partial \vec{\mathbf{M}}}{\partial T}\right)_{V,H} \cdot \left[\frac{\partial \vec{\mathbf{H}}}{\partial t} + (\vec{\mathbf{q}}\cdot\nabla)\vec{\mathbf{H}}\right] + mN\,C_{pt}\left(\in\frac{\partial}{\partial t} + \vec{\mathbf{q}}_d\cdot\nabla\right)T = \chi_T\,\nabla^2 T + \delta(\nabla\times\vec{\mathbf{N}})\cdot\nabla T \qquad ...(4)$$

The equation of state is given by

$$\rho = \rho_0\left[1 - \alpha(T - T_a)\right] \qquad ...(5)$$

The equation of motion and continuity of the dust particles are given by

$$mN\left[\frac{\partial}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}_d\cdot\nabla)\right]\vec{\mathbf{q}}_d = KN(\vec{\mathbf{q}} - \vec{\mathbf{q}}_d) \qquad ...(6)$$

and

$$\in\frac{\partial N}{\partial t} + \nabla\cdot(N\,\vec{\mathbf{q}}_d) = 0 \qquad ...(7)$$

Where $\vec{\mathbf{q}}, \vec{\mathbf{N}}, \vec{\mathbf{q}}_d, p, t, \rho, \rho_0, \mu, \zeta, \kappa_1, \alpha', \beta', \gamma', \mu_0, j, \vec{\mathbf{H}}, \vec{\mathbf{B}}, \vec{\mathbf{M}}, T, \rho_s, C_{V,H},$ $mN, C_{pt}, K = 6\pi\mu r, \chi_T, \delta, \alpha, T_a$ and $\hat{\mathbf{e}}_z$ denote respectively filter velocity, micro-rotation, velocity of dust particles, pressure, time, density, reference density, viscosity, coupling viscosity coefficient, medium permeability, micro-polar viscosity coefficients, magnetic permeability for free space ($\mu_0 = 4\pi\times 10^{-7}$ Henry m^{-1}), micro-inertia coefficient, magnetic field, magnetic induction, magnetization, temperature, density of solid matrix, specific heat at constant volume and magnetic field, specific heat of solid matrix, mass of the particles per unit volume, specific heat of dust particles, stokes drag coefficient, thermal conductivity, micro-polar heat conduction coefficient, coefficient of thermal expansion, average temperature, which is defined as $T_a = \frac{T_0 + T_1}{2}$, and unit vector along *z*-axis.

Also $N = N(\bar{x}, t)$ where $\bar{x} = (x, y, z)$, denotes the number density of the dust particles. T_0 and T_1 are constant temperatures at lower and upper boundaries respectively. The partial derivatives are material properties that can be evaluated once the magnetic equation of state is known.

In ferro-hydro-dynamics the free charge and the electric displacement are assumed to be absent, therefore the Maxwell's equation yield

$$\nabla\cdot\vec{\mathbf{B}} = 0, \qquad \nabla\times\vec{\mathbf{H}} = \mathbf{0} \qquad ...(8)$$

In **Chu** formulation of electrodynamics, the magnetic field, magnetization and magnetic induction are related by

$$\vec{\mathbf{B}} = \mu_0(\vec{\mathbf{H}} + \vec{\mathbf{M}}) \qquad ...(9)$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature, so that

$$\vec{\mathbf{M}} = \frac{\vec{\mathbf{H}}}{H} M(H, T) \qquad ...(10)$$

The magnetic equation of state is linearlized about H_o and T_a, which is given by

$$M = M_o + \chi(H - H_o) - K_2(T - T_a) \qquad ...(11)$$

Where $\vec{\mathbf{H}} = (0, 0, H_o^{ext})$ i.e., $\vec{\mathbf{H}} = H_o^{ext}\,\hat{\mathbf{e}}_z$, $\hat{\mathbf{e}}_z$ is the unit vector along *z*-axis and H_o is the uniform magnetic field of the fluid layer when placed in an external magnetic field $\vec{\mathbf{H}}$, and $\vec{\mathbf{B}}$ is the magnetic induction.

Now $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_o, T_a}$ denotes the magnetic susceptibility.

$K_2 = -\left(\frac{\partial M}{\partial T}\right)_{H_o, T_a}$ denotes the pyromagnetic coefficient, and $H = |\vec{\mathbf{H}}|$, $M = |\vec{\mathbf{M}}|$ and $M_o = M(H_o, T_a)$

III. BASIC STATE OF THE PROBLEM

The basic state is assumed to be quiescent state which is given by

$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b = (0,0,0), \vec{\mathbf{N}} = \vec{\mathbf{N}}_b = (0,0,0), \vec{\mathbf{q}}_d = (\vec{\mathbf{q}}_d)_b = (0,0,0)$, $\rho = \rho_b(z),\ p = p_b(z), N = N_b$

$\vec{\mathbf{H}} = \vec{\mathbf{H}}_b = \vec{\mathbf{H}}_b(z),\ \vec{\mathbf{M}} = \vec{\mathbf{M}}_b = \vec{\mathbf{M}}_b(z),\ \vec{\mathbf{B}} = \vec{\mathbf{B}}_b$ ...(12)

Using this basic state equations (1) to (11) yield

$$-\nabla(\vec{\mathbf{H}}_b . \vec{\mathbf{B}}_b) + \frac{dp_b}{dz} + \rho_b g = 0 \quad ...(13)$$

$$T = T_b(z) = -\beta z + T_a,\ \beta = \frac{T_1 - T_0}{d} \quad ...(14)$$

$$\rho = \rho_b = \rho_0(1 + \alpha\beta z) \quad ...(15)$$

$$N = N_b = N_o \quad ...(16)$$

From (11), we have

$$M_b = M_o + \chi(H_b - H_o) - K_2(T_b - T_a) \quad ...(17)$$

From (8) and (9), we have

$$\nabla . \vec{B} = 0 \Rightarrow \mu_0 \nabla .(\vec{\mathbf{H}} + \vec{\mathbf{M}}) = 0$$

$$\Rightarrow \vec{\mathbf{H}} + \vec{\mathbf{M}} = (H_o + M_0)\hat{\mathbf{e}}_z \quad ...(18)$$

For basic state, we have

$$\vec{\mathbf{H}}_b + \vec{\mathbf{M}}_b = (H_o + M_o)\hat{\mathbf{e}}_z$$

or $\quad M_b = M_o - h_b$ ...(19)

From (17) and (19), we get

$$M_b = \left[M_o - \frac{K_2(T_b - T_a)}{1+\chi}\right] \text{ or } \vec{\mathbf{M}}_b = \left[M_o - \frac{K_2(T_b - T_a)}{1+\chi}\right]\hat{\mathbf{e}}_z \quad ...(20)$$

and $$H_b = \left[H_o + \frac{K_2(T_b - T_a)}{1+\chi}\right] \text{ or } \vec{\mathbf{H}}_b = \left[H_o + \frac{K_2(T_b - T_a)}{1+\chi}\right]\hat{\mathbf{e}}_z \quad ...(21)$$

and $$H_o + M_o = H_o^{ext} \text{ and } \vec{\mathbf{B}}_b = \mu_0(H_o + M_o)\hat{\mathbf{e}}_z \quad ...(22)$$

IV. PERTURBATION EQUATIONS

Let $\vec{\mathbf{q}}', \vec{\mathbf{N}}', \vec{\mathbf{q}}_1', p', \rho', \theta, \vec{\mathbf{H}}', \vec{\mathbf{M}}'$ and N' denote respectively the small perturbations in $\vec{\mathbf{q}}, \vec{\mathbf{N}}, \vec{\mathbf{q}}_d, p, \rho, T, \vec{\mathbf{H}}, \vec{\mathbf{M}}$ and N, therefore the new variable become

$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b + \vec{\mathbf{q}}' = \vec{\mathbf{q}}',\ \vec{\mathbf{N}} = \vec{\mathbf{N}}_b + \vec{\mathbf{N}}' = \vec{\mathbf{N}}',$

$\vec{\mathbf{q}}_d = (\vec{\mathbf{q}}_d)_b + \vec{\mathbf{q}}_1' = \vec{\mathbf{q}}_1',\ p = p_b + p',\ \rho = \rho_b + \rho',\ T = T_b + \theta$

$\vec{\mathbf{H}} = \vec{\mathbf{H}}_b + \vec{\mathbf{H}}',\ \vec{\mathbf{M}} = \vec{\mathbf{M}}_b + \vec{\mathbf{M}}',\ \vec{\mathbf{B}} = \vec{\mathbf{B}}_b + \vec{\mathbf{B}}'$ and

$N = N_b + N'$

Using above perturbations and equations (13), (14), (15), (16), (20), (21), (22), equations (1) to (11) become

$$\nabla . \vec{\mathbf{q}}' = 0 \quad ...(23)$$

$$\frac{\rho_o}{\in}\left[\frac{\partial \vec{\mathbf{q}}'}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{q}}'\right] = -\nabla p' - \rho' g\,\hat{\mathbf{e}}_z - \frac{(\mu+\zeta)}{\kappa_1}\vec{\mathbf{q}}' + \frac{K(N_0 + \mathbf{N}')}{\in}(\vec{\mathbf{q}}_1' - \vec{\mathbf{q}}') + (\vec{\mathbf{H}}_b.\nabla)\vec{\mathbf{B}}' + (\vec{\mathbf{B}}_b'.\nabla)\vec{\mathbf{H}}' + (\vec{\mathbf{B}}'.\nabla)\vec{\mathbf{H}}_b + \vec{\mathbf{H}}_b \times (\nabla \times \vec{\mathbf{B}}')$$
$$+ (\vec{\mathbf{H}}'.\nabla)\vec{\mathbf{B}}' + (\vec{\mathbf{B}}'.\nabla)\vec{\mathbf{H}}' + \vec{\mathbf{H}}' \times (\nabla \times \vec{\mathbf{B}}') + \frac{2\rho_o}{\in}(\vec{\mathbf{q}}' \times \vec{\mathbf{\Omega}}) + \zeta(\nabla \times \vec{\mathbf{N}}') \quad ...(24)$$

$$\rho_o\, j\left[\frac{\partial \vec{\mathbf{N}}'}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{N}}'\right] = (\alpha' + \beta')\nabla(\nabla.\vec{\mathbf{N}}') + \gamma'\nabla^2\vec{\mathbf{N}}' + \zeta\left(\frac{1}{\in}\nabla \times \vec{\mathbf{q}}' - 2\vec{\mathbf{N}}'\right) + \mu_0\left[(\vec{\mathbf{M}}_b + \vec{\mathbf{M}}') \times (\vec{\mathbf{H}}_b + \vec{\mathbf{H}}')\right] \quad ...(25)$$

$$\left[\rho_o C_{V,H} - \mu_0(\vec{\mathbf{H}}_b + \vec{\mathbf{H}}').\left(\frac{\partial(\vec{\mathbf{M}}_b + \vec{\mathbf{M}}')}{\partial(T_b + \theta)}\right)_{V,H}\right]\left[\in\frac{\partial(T_b + \theta)}{\partial t} + (\vec{\mathbf{q}}'.\nabla)(T_b + \theta)\right]$$
$$+ (1-\in)\rho_s C_s \frac{\partial(T_b + \theta)}{\partial t} + \mu_0(T_b + \theta)\left(\frac{\partial(\vec{\mathbf{M}}_b + \vec{\mathbf{M}}')}{\partial(T_b + \theta)}\right)_{V,H}.\left[\in\frac{\partial(\vec{\mathbf{H}}_b + \vec{\mathbf{H}}')}{\partial t} + (\vec{\mathbf{q}}'.\nabla)(\vec{\mathbf{H}}_b + \vec{\mathbf{H}}')\right]$$
$$+ m(N_b + N')C_{pt}\left(\in\frac{\partial}{\partial t} + (\vec{\mathbf{q}}_1'.\nabla)\right)(T_b + \theta) = \chi_T \nabla^2(T_b + \theta) + \delta(\nabla \times \vec{\mathbf{N}}').\nabla(T_b + \theta) \quad ...(26)$$

$$\rho' = -\rho_o \alpha\theta \quad ...(27)$$

$$\vec{\mathbf{q}}_1' = \frac{1}{\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)}\vec{\mathbf{q}}' = \frac{1}{L_0}\vec{\mathbf{q}}' \quad ...(28)$$

$N' = a$ constant (not a function of time) ...(29)

From equations (10) and (11), we have

$$(H_b + H')(\vec{\mathbf{M}}_b + \vec{\mathbf{M}}') = (\vec{\mathbf{H}}_b + \vec{\mathbf{H}}')\left[M_o + \chi H' + \chi(H_b - H_o) - K_2\theta - K_2(T_b - T_a)\right]$$

Taking components along *x, y,* and *z*-directions respectively, we have

Along *x*-direction :

$$H_1' + M_1' = \left(1+\frac{M_0}{H_0}\right)H_1' \quad ...(30)$$

Along *y*-direction :

$$H_2' + M_2' = \left(1+\frac{M_0}{H_0}\right)H_2' \quad ...(31)$$

Along *z*-direction :

$$H_3' + M_3' = (1+\chi)H_3' - K_2\theta \quad ...(32)$$

Using (28), equation (26) can be rewritten as

$$\in\left[\rho_o C_{V,H} - \mu_0(\vec{\mathbf{H}}_b+\vec{\mathbf{H}}').\left[\frac{\partial\vec{\mathbf{M}}_b}{\partial(T_b+\theta)}\right]_{V,H} - \mu_0(\vec{\mathbf{H}}_b+\vec{\mathbf{H}}').\left[\frac{\partial\vec{\mathbf{M}}'}{\partial(T_b+\theta)}\right]_{V,H}\right]\times\left[\frac{\partial\theta}{\partial t}+(\vec{\mathbf{q}}'.\nabla)T_b+(\vec{\mathbf{q}}'.\nabla)\theta\right]+(1-\in)\rho_s C_s\frac{\partial\theta}{\partial t}$$

$$+\left[\mu_0(T_b+\theta)\left(\frac{\partial\vec{\mathbf{M}}_b}{\partial(T_b+\theta)}\right)_{V,H}+\mu_0(T_b+\theta)\left(\frac{\partial\vec{\mathbf{M}}'}{\partial(T_b+\theta)}\right)_{V,H}\right].\left[\frac{\partial\vec{\mathbf{H}}'}{\partial t}+(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{H}}_b+(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{H}}'\right]$$

$$+m(N_b+N')C_{pt}\left[\in\frac{\partial}{\partial t}+\frac{1}{L_0}(\vec{\mathbf{q}}'.\nabla)\right](T_b+\theta)$$

$$=\chi_T\nabla^2\theta+\delta(\nabla\times\vec{\mathbf{N}}').\nabla T_b+\delta(\nabla\times\vec{\mathbf{N}}').\nabla\theta \quad ...(33)$$

In order to make linear, we ignore the terms $(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{q}}'$, $N'(\vec{\mathbf{q}}_1'-\vec{\mathbf{q}}')$, $(\vec{\mathbf{M}}'.\nabla)\vec{\mathbf{H}}'$, $(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{N}}'$, $(\vec{\mathbf{H}}'.\nabla)\vec{\mathbf{B}}'$, $\vec{\mathbf{M}}'\times\vec{\mathbf{H}}'$, $\vec{\mathbf{H}}'.\left(\frac{\partial\vec{\mathbf{M}}'}{\partial(T_b+\theta)}\right)_{V,H}$, $(\vec{\mathbf{q}}'.\nabla)\theta$, $\theta\left(\frac{\partial\vec{\mathbf{M}}'}{\partial(T_b+\theta)}\right)_{V,H}$, $(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{H}}'$, $N'(\vec{\mathbf{q}}',\nabla)$, $N'\frac{\partial\theta}{\partial t}$, $(\nabla\times\vec{\mathbf{N}}').\nabla\theta$, $(\vec{\mathbf{B}}'.\nabla)\vec{\mathbf{H}}'$, $\vec{\mathbf{H}}'\times(\nabla\times\vec{\mathbf{B}}')$, we obtain

$$\nabla.\vec{\mathbf{q}}' = 0 \quad ...(34)$$

$$\left[\frac{L_0\rho_0}{\in}+\frac{mN_0}{\in}\right]\frac{\partial\vec{\mathbf{q}}'}{\partial t} = L_0\left[-\nabla p' - \rho' g\hat{\mathbf{e}}_z - \left(\frac{\mu+\zeta}{\kappa_1}\right)\vec{\mathbf{q}}' + \frac{2\rho_0\Omega_0}{\in}(\vec{\mathbf{q}}'\times\hat{\mathbf{e}}_z)\right.$$

$$\left.+\zeta\nabla\times\vec{\mathbf{N}}' + \mu_0(H_0+M_0)\frac{\partial\vec{\mathbf{H}}'}{\partial z} - \frac{\mu_0 K_2\beta\hat{\mathbf{e}}_z}{(1+\chi)}\left[(1+\chi)H_3' - K_2\theta\right]\right] \quad ...(35)$$

$$\rho_0 j\frac{\partial\vec{\mathbf{N}}'}{\partial t} = (\alpha'+\beta'+\gamma')\nabla(\nabla.\vec{\mathbf{N}}') - \gamma'(\nabla\times\nabla\times\vec{\mathbf{N}}') + \zeta\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}}' - 2\vec{\mathbf{N}}'\right) + \mu_0(\vec{\mathbf{M}}_b\times\vec{\mathbf{H}}' + \vec{\mathbf{M}}'\times\vec{\mathbf{H}}_b) \quad ...(36)$$

$$L_0\left[(C_1+mN_0\in C_{pt})\frac{\partial\theta}{\partial t} - \mu_0 T_0 K_2\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial z}\right)\right] = L_0\left[\chi_T\nabla^2\theta + \left\{C_2\beta - \frac{\mu_0 T_0 K_2^2\beta}{1+\chi}\right\}w - \beta\delta\xi\right] + mN_0\beta C_{pt}w \quad ...(37)$$

Where $C_1 = \in\rho_0 C_{V,H} + (1-\in)\rho_s C_s + \in\mu_0 K_2 H_0$, $C_2 = \in\rho_0 C_{V,H} + \in\mu_0 K_2 H_0$,

$\vec{\mathbf{B}}' = (B_1', B_2', B_3')$, $\vec{\mathbf{H}}' = (H_1', H_2', H_3')$, $\xi = (\nabla\times\vec{\mathbf{N}}').\hat{\mathbf{e}}_z$, $W = (\vec{\mathbf{q}}'.\hat{\mathbf{e}}_z)$

V. DISPERSION RELATIONS

Taking curl twice on both sides of equation (35), and taking z-component, we get

$$\left[\left\{\frac{\rho_0}{\in}\frac{\partial}{\partial t}+\left(\frac{\mu+\zeta}{\kappa_1}\right)\right\}L_0+\frac{mN_0}{\in}\frac{\partial}{\partial t}\right]\nabla^2 w = L_0\left[\rho_0\alpha g\nabla_1^2\theta+\zeta\nabla^2\xi-\frac{2\rho_0\Omega_0}{\in}\frac{\partial\zeta_z}{\partial z}-\mu_0K_2\beta\nabla_1^2\left(\frac{\partial\phi'}{\partial z}\right)+\frac{\mu_0K_2^2\beta}{1+\chi}\nabla_1^2\theta\right] \quad ...(38)$$

Taking curl once on both sides of (36), and taking z-component, we get

$$\rho_0 j\frac{\partial\xi}{\partial t}=\gamma'\nabla^2\xi-\rho\left(\frac{1}{\in}\nabla^2 w+2\xi\right) \quad ...(39)$$

Taking curl once on both sides of equation (35), and taking z-component, we get

$$\left[\left\{\frac{\rho_0}{\in}\frac{\partial}{\partial t}+\left(\frac{\mu+\zeta}{\kappa_1}\right)\right\}L_0+\frac{mN_0}{\in}\frac{\partial}{\partial t}\right]\zeta_z=\frac{2\rho_0L_0\Omega_0}{\in}\frac{\partial w}{\partial z}+L_0\zeta(\nabla\times\nabla\times\vec{\mathbf{N}}').\hat{\mathbf{e}}_z \quad ...(40)$$

Taking curl twice on both sides of equation (36) and taking *z*-component, we get

$$\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\nabla^2+2\zeta\right](\nabla\times\nabla\times\vec{\mathbf{N}}').\hat{\mathbf{e}}_z=-\frac{\zeta}{\in}\nabla^2\zeta_z \quad ...(41)$$

Eliminating $(\nabla\times\nabla\times\vec{\mathbf{N}}').\hat{\mathbf{e}}_z$ between (40) and (41), we get

$$\left[\left\{\frac{\rho_0}{\in}\frac{\partial}{\partial t}+\left(\frac{\mu+\zeta}{\kappa_1}\right)\right\}L_0+\frac{mN_0}{\in}\frac{\partial}{\partial t}\right]\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\nabla^2+2\zeta\right]\zeta_z=\frac{2\rho_0L_0\Omega_0}{\in}\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\nabla^2+2\zeta\right]\frac{\partial w}{\partial z}-L_0\frac{\zeta^2}{\in}\nabla^2\zeta_z \quad ...(42)$$

Where $\zeta_z=(\nabla\times\vec{\mathbf{q}}').\hat{\mathbf{e}}_z$

From equations (30), (31) and (32), we get

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^2\phi'+(1+\chi)\frac{\partial^2\phi'}{\partial z^2}-K_2\frac{\partial\theta}{\partial z}=0 \quad ...(43)$$

Second equation of equation (8) implies that in the absence of electric current and electromagnetic induction, the field $\vec{\mathbf{H}}$ can be regarded as conservative, so we may take $\vec{\mathbf{H}}'=\nabla\phi'$, where ϕ' is the perturbed magnetic potential.

VI. NORMAL MODE ANALYSIS

Now we analyze the perturbations w,θ,ξ,ζ_z and ϕ' into two-dimensional periodic waves by considering the following form

$$[w,\theta,\xi,\zeta_z,\phi']=[W(z,t),\Theta(z,t),X(z,t),G(z,t),\Phi(z,t)]\exp(ixk_x+iyk_y) \quad ...(44)$$

Where $a=\sqrt{k_x^2+k_y^2}$ is the wave number, $\nabla^2\equiv\left(\frac{\partial^2}{\partial z^2}-a^2\right)$ and $\nabla_1^2\equiv -a^2$.

Using above normal mode analysis, equations (37), (38), (39), (42), and (43) become

$$\left[\left(\frac{\rho_0}{\in}\frac{\partial}{\partial t}+\frac{\mu+\zeta}{\kappa_1}\right)L_0+\frac{mN_0}{\in}\frac{\partial}{\partial t}\right]\left(\frac{\partial^2}{\partial z^2}-a^2\right)W=L_0\left[-\rho_0\alpha ga^2\Theta+\zeta\left(\frac{\partial^2}{\partial z^2}-a^2\right)X-\frac{2\rho_0\Omega_0}{\in}\frac{\partial G}{\partial z}+\mu_0K_2\beta a^2\frac{\partial\Phi}{\partial z}-\frac{\mu_0K_2^2\beta a^2\Theta}{1+\chi}\right] \quad ...(45)$$

$$\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\left(\frac{\partial^2}{\partial z^2}-a^2\right)+2\zeta\right]X=-\frac{\zeta}{\in}\left(\frac{\partial^2}{\partial z^2}-a^2\right)W \quad ...(46)$$

$$\left[\left(\frac{\rho_0}{\in}\frac{\partial}{\partial t}+\frac{\mu+\zeta}{\kappa_1}\right)L_0+\frac{mN_0}{\in}\frac{\partial}{\partial t}\right]\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\left(\frac{\partial^2}{\partial z^2}-a^2\right)+2\zeta\right]G=\frac{2\rho_0L_0\Omega_0}{\in}\left[\rho_0 j\frac{\partial}{\partial t}-\gamma'\left(\frac{\partial^2}{\partial z^2}-a^2\right)+2\zeta\right]\frac{\partial W}{\partial z}-\frac{L_0\zeta^2}{\in}\left(\frac{\partial^2}{\partial z^2}-a^2\right)G \quad ...(47)$$

$$\left(1+\frac{M_0}{H_0}\right)(-a^2)\Phi+(1+\chi)\frac{\partial^2\Phi}{\partial z^2}-K_2\frac{\partial\Theta}{\partial z}=0 \quad ...(48)$$

and

$$L_0\left[(C_1+mN_0\in C_{pt}]\frac{\partial\Theta}{\partial t}-\mu_0T_0K_2\frac{\partial}{\partial t}\left(\frac{\partial\Phi}{\partial z}\right)\right]=L_0\left[\chi_T\left(\frac{\partial^2}{\partial z^2}-a^2\right)\Theta+\left\{C_2\beta-\frac{\mu_0T_0K_2^2\beta}{1+\chi}\right\}W-\beta\delta X\right]+mN_0\beta C_{pt}W \quad ..(49)$$

Now converting the equations (45), (46), (47), (48) and (49) into non-dimensional form by using the following non-dimensional parameters and non-dimensional quantities

$$t^*=\frac{vt}{d^2},\ W^*=\frac{d}{v}W,\ Z^*=\frac{z}{d},\ D\equiv\frac{\partial}{\partial z^*},\ K_1^*=\frac{K_1}{d^2},\ a^*=ad,\ X^*=\frac{Xd^3}{v},\ G^*=\frac{d^2}{v}G$$

$$\Theta^*=\frac{\chi_Ta^*R^{1/2}}{C_2\beta vd}\Theta,\ \Phi^*=\frac{(1+\chi)\chi_Ta^*R^{1/2}}{K_2C_2\beta vd^2}\Phi,\ R=\frac{g\alpha\beta d^4C_2}{v\chi_T},\ P_r=\frac{vC_2}{\chi_T},\ P_r'=\frac{vC_1}{\chi_T},\ N_1=\frac{\zeta}{\mu}$$

$$N_2=\frac{\gamma'}{\mu d^2},\ N_3=\frac{\delta}{C_2d^2},\ \bar{j}=\frac{j}{d^2},\ M_1=\frac{\mu_0K_2^2\beta}{(1+\chi)\alpha\rho_0 g},\ M_2=\frac{\mu_0T_0K_2^2}{(1+\chi)C_2},\ M_3=\frac{\left(1+\frac{M_0}{H_0}\right)}{(1+\chi)}$$

$$\tau = \frac{m\nu}{Kd^2},\ L_0^* = \left(\tau \frac{\partial}{\partial t^*} + 1\right),\ f = \frac{mN_0}{\rho_0},\ h = \frac{mN_0 C_{pt}}{C_2},\ T_A = \left(\frac{2\Omega_0 d^2}{\nu}\right)^2$$

We have

$$\left[L_0^*\left(\frac{1}{\in}\frac{\partial}{\partial t^*} + \frac{1+N_1}{K_1^*}\right) + \frac{f}{\in}\frac{\partial}{\partial t^*}\right](D^2 - a^{*2})W^* = L_0^*\left[\left\{M_1 D\Phi^* - (1+M_1)\Theta^*\right\}a^* R^{1/2} + N_1(D^2 - a^{*2})X^* - \frac{T_A^{1/2}}{\in}DG^*\right] \quad ...(50)$$

$$\left[\bar{j}\frac{\partial}{\partial t^*} - N_2(D^2 - a^{*2}) + 2N_1\right]X^* = -\frac{N_1}{\in}(D^2 - a^{*2})W^* \quad ...(51)$$

$$\left[L_0^*\left(\frac{1}{\in}\frac{\partial}{\partial t^*} + \frac{1+N_1}{K_1^*}\right) + \frac{\delta}{\in}\frac{\partial}{\partial t^*}\right]\left[\bar{j}\frac{\partial}{\partial t^*} - N_2(D^2 - a^{*2}) + 2N_1\right]G^* = \frac{L_0^* T_A^{1/2}}{\in}\left[\bar{j}\frac{\partial}{\partial t^*} - N_2(D^2 - a^{*2}) + 2N_1\right]DW^* - \frac{L_0^* N_1^2}{\in}(D^2 - a^{*2})G^* \quad ...(52)$$

$$D^2\Phi^* - a^{*2}M_3\Phi^* - D\Theta^* = 0 \quad ...(53)$$

$$L_0^*\left[(P_r' + \in hP_r)\frac{\partial\Theta^*}{\partial t^*} - M_2 P_r \frac{\partial}{\partial t^*}(D\Phi^*)\right] = L_0^*(D^2 - a^{*2})\Theta^* + \left\{L_0^*(1-M_2) + h\right\}a^* R^{1/2}W^* - a^* R^{1/2} L_0^* N_3 X^* \quad ...(54)$$

In equations (50) to (54) $\nu = \frac{\mu}{\rho_0}$ denotes the kinematic viscosity of the fluid, M_1, M_3 and h denote respectively buoyancy magnetization, non-buoyancy magnetization and dust particle parameters. $R, T_A, P_r, P_r', N_1, N_3$ and $\bar{j}$ denote respectively thermal Rayleigh number, Taylor number, Prandtl number, coupling parameter, heat conduction parameter and microinertia parameter.

VII. EXACT SOLUTION FOR FREE BOUNDARIES

In this problem, we consider that both the boundaries are free as well as perfect conductor of heat. The case of two free boundaries is of little physical interest but mathematically, it is important because in case of free boundaries one can derive an exact solution. Here we consider the case of an infinite magnetic susceptibility (i.e., $\chi \to \infty$) and we neglect the deformability of horizontal surfaces. The exact solution of equations (50)-(54) subject to the boundary conditions

$$W^* = D^2W^* = \Theta^* = D\Phi^* = X^* = DG^* = 0 \text{ at } Z^* = -\frac{1}{2} \text{ and } \frac{1}{2} \quad ...(55)$$

is written in the form

$$\left.\begin{aligned} &W^* = P_1 e^{\sigma t^*}\cos\pi z^* \qquad \Theta^* = P_2 e^{\sigma t^*}\cos\pi z^* \\ &D\Phi^* = P_3 e^{\sigma t^*}\cos\pi z^* \text{ or } \Phi^* = \left(\frac{P_3}{\pi}\right)e^{\sigma t^*}\sin\pi z^* \text{ and } X^* = P_4 e^{\sigma t^*}\cos\pi z^* \\ &\text{and } DG^* = P_5 e^{\sigma t^*}\cos\pi z^* \qquad \text{or } G^* = \left(\frac{P_5}{\pi}\right)e^{\sigma t^*}\sin\pi z^* \end{aligned}\right\} \quad ...(56)$$

Where P_1, P_2, P_3, P_4, P_5 are constants and σ is the growth rate, which is in general, a complex number. Substituting eq. (56) in eqs. (50)-(54) and dropping asterisks for convenience, we get

$$\left[(1+\tau\sigma)\left(\frac{\sigma}{\in} + \frac{1+N_1}{K_1}\right) + \frac{f}{\in}\sigma\right](\pi^2 + a^2)P_1 - (1+\pi\sigma)(1+M_1)P_2 a R^{1/2}$$

$$+M_1(1+\tau\sigma)P_3 aR^{1/2} - N_1(\pi^2 + a^2)P_4(1+\tau\sigma) - \frac{T_A^{1/2}}{\in}P_5(1+\tau\sigma) = 0 \quad ...(57)$$

$$\frac{N_1}{\in}(\pi^2 + a^2)P_1 - \left[\bar{j}\sigma + N_2\left(\pi^2 + a^2\right) + 2N_1\right]P_4 = 0 \quad ...(58)$$

$$\frac{\pi^2(1+\tau\sigma)T_A^{1/2}}{\in}\left[\bar{j}\sigma + N_2(\pi^2 + a^2) + 2N_1\right]P_1 + \left\{\left[(1+\tau\sigma)\left(\frac{\sigma}{\in} + \frac{1+N_1}{K_1}\right) + \frac{f}{\in}\sigma\right] \times \left[\bar{j}\sigma + N_2(\pi^2 + a^2) + 2N_1\right] - \frac{(1+\tau\sigma)N_1^2}{\in}(\pi^2 + a^2)\right\}P_5 = 0 \quad ...(59)$$

$$\pi^2 P_2 - (\pi^2 + a^2 M_3)P_3 = 0 \quad ...(60)$$

$$aR^{1/2}\left\{(1+\tau\sigma)(1-M_2) + h\right\}P_1 - (1+\tau\sigma)\left\{\sigma(P_r' + \in hP_r) + (\pi^2 + a^2)\right\}P_2 + M_2 P_r \sigma(1+\tau\sigma)P_3 - aR^{1/2}(1+\tau\sigma)N_3 P_4 = 0 \quad ...(61)$$

For the existence of non-trivial solutions of equations (57)-(61), the determinant of the coefficients of P_1, P_2, P_3, P_4 and P_5 must vanish. Thus we have

$$-\frac{\pi^6 T_{A_1}(1+\tau_1 i\sigma_1)^2}{\epsilon^2}\left[\bar{j}_1 i\sigma_1 + N_2'(1+x_1)+2N_1\right]^2$$

$$\times\left\{\pi^4 M_2 P_r\, i\sigma_1(1+\tau_1 i\sigma_1) - \pi^4(1+x_1 M_3)(1+\tau_1 i\sigma_1)\left[i\sigma_1(P_r' + \epsilon h P_r)+1+x_1\right]\right\}$$

$$+\frac{\pi^4 N_1(1+x_1)}{\epsilon}\left\{\left[(1+\tau_1 i\sigma_1)\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\left[\bar{j}_1 i\sigma_1 + N_2'(1+x_1)+2N_1\right]\right.$$

$$\left.-(1+\tau_1 i\sigma_1)\frac{\pi^2 N_1^2(1+x_1)}{\epsilon}\right\}.\left\{-x_1 R_1 \pi^4 M_1 N_3'(1+\tau_1 i\sigma_1)^2+\pi^4 M_2 P_r\, i\sigma_1 N_1(1+x_1)(1+\tau_1 i\sigma_1)^2\right\}$$

$$+\pi^4(1+x_1 M_3)\frac{N_1(1+x_1)}{\epsilon}\left\{\left[(1+\tau_1 i\sigma_1\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\left[\bar{j}\, i\sigma_1+N_2'(1+x_1)+2N_1\right]\right.$$

$$\left.-(1+\tau_1 i\sigma_1)\frac{\pi^2 N_1^2(1+x_1)}{\epsilon}\right\}$$

$$\times\left\{\pi^4 x_1 R_1 N_3'(1+M_1)(1+\tau_1 i\sigma_1)^2-\pi^4 N_1(1+x_1)(1+\tau_1 i\sigma_1)^2\left[i\sigma_1(P_r'+\epsilon h P_r)+1+x_1\right]\right\}$$

$$-\pi^2\left[\bar{j}\, i\sigma_1+N_2'(1+x_1)+2N_1\right]\left\{\left[(1+\tau_1 i\sigma_1)\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\right.$$

$$\left.\times\left[\bar{j}_1 i\sigma_1+N_2'(1+x_1)+2N_1\right]-(1+\tau_1 i\sigma_1)\frac{\pi^2 N_1^2(1+x_1)}{\epsilon}\right\}$$

$$\times\left\{\pi^4(1+x_1)M_2 P_r\, i\sigma_1(1+\tau_1 i\sigma_1)\left[(1+\tau_1 i\sigma_1)\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\right.$$

$$\left.-\pi^6 x_1 R_1 M_1(1+\tau_1 i\sigma_1)\left[(1+\tau_1 i\sigma_1)(1-M_2)+h\right]\right\}$$

$$-\pi^2(1+x_1 M_3)\left[\bar{j}_1 i\sigma_1+N_2'(1+x_1)+2N_1\right]\left\{\left[(1+\tau_1 i\sigma_1)\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\right.$$

$$\left.\times\left[\bar{j}_1 i\sigma_1+N_2'(1+x_1)+2N_1\right]-(1+\tau_1 i\sigma_1)\frac{\pi^2 N_1^2(1+x_1)}{\epsilon}\right\}$$

$$\times\left\{-\pi^2(1+x_1)(1+\tau_1 i\sigma_1)\left[(1+\tau_1 i\sigma_1)\left(\frac{\pi^2}{\epsilon}i\sigma_1+\frac{\pi^2(1+N_1)}{K_1'}\right)+\frac{\pi^2 f}{\epsilon}i\sigma_1\right]\right.$$

$$\left.\times\pi^2\left[i\sigma_1(P_r'+\epsilon h P_r)+1+x_1\right]+\pi^6 x_1 R_1(1+M_1)(1+\tau_1 i\sigma_1)\left[(1+\tau_1 i\sigma_1)(1-M_2)+h\right]\right\}=0\,. \quad ..(62)$$

Where $i\sigma_1=\frac{\sigma}{\pi^2},\ x_1=\frac{a^2}{\pi^2},\ R_1=\frac{R}{\pi^4},\ \bar{j}_1=\pi^2\bar{j},\ N_2'=\pi^2 N_2,\ N_3'=\pi^2 N_3,\ K_1'=\pi^2 K_1,\ T_{A_1}=\frac{T_A}{\pi^4},\ \tau_1=\pi^2\tau$

Here we assume that

$b=1+x_1,\ L_1=N_2'(1+x_1)+2N_1=N_2'b+2N_1,\ L_2=(1+x_1 M_3),\ L_3=\left[\frac{\tau_1(1+N_1)}{K_1'}+\frac{1+f}{\epsilon}\right], L_4=\frac{1+N_1}{K_1'},$

$L_5=P_r'+\epsilon h P_r$, we have

$$A_0\sigma_1^8+iA_1\sigma_1^7+A_2\sigma_1^6+iA_3\sigma_1^5+A_4\sigma_1^4+iA_5\sigma_1^3+A_6\sigma_1^2+iA_7\sigma_1+A_8=0 \quad ...(63)$$

Where $A_0=\frac{L_2\bar{j}_1^2\tau_1^3 b L_5}{\epsilon^2}-\frac{\bar{j}_1 b M_2 P_r \tau_1^2}{\epsilon}$

$$
\begin{aligned}
A_1 = & -\frac{L_1 L_2 \bar{j}_1 \tau_1^3 b L_5}{\in^2} - \frac{L_2 \bar{j}_1^2 \tau_1}{\in}\left(\frac{b\tau_1 L_5}{\in} + b\tau_1 L_3 L_5 + \frac{b^2\tau_1^2}{\in}\right) - \frac{L_2 \bar{j}_1 b \tau_1^2 L_5}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right) \\
& + \frac{\bar{j}_1^2 \tau_1}{\in}\left(\frac{bM_2 P_r \tau_1}{\in} + bM_2 P_r L_3 \tau_1\right) + \frac{L_1 b M_2 P_r \tau_1^2}{\in} + \frac{\bar{j}_1 b M_2 P_r \tau_1^2}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)
\end{aligned}
$$

$$
\begin{aligned}
A_2 = & \frac{T_{A_1}\tau_1^2 \bar{j}_1}{\in^2}\left(M_2 P_r \tau_1 - L_2 L_5 \tau_1\right) - \frac{\bar{j}_1 \tau_1^3 M_2 P_r N_1^2 b^2}{\in^2} + \frac{L_2 \bar{j}_1 \tau_1^3 N_1^2 b^2 L_5}{\in^2} - \frac{L_1 L_2 \bar{j}_1 \tau_1}{\in}\left(\frac{b\tau_1 L_5}{\in} + b\tau_1 L_3 L_5 + \frac{b^2\tau_1^2}{\in}\right) \\
& - \frac{L_1 L_2 b \tau_1^2 L_5}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right) + \frac{bM_2 P_r \tau_1^2}{\in}\left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right) - \frac{L_2 \bar{j}_1 b \tau_1^2 L_5}{\in}\left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right) \\
& - \frac{L_2 \bar{j}_1^2 \tau_1}{\in}\left(\begin{array}{l} bL_3 L_5 + \dfrac{\tau_1 b^2}{\in} + b\tau_1 L_4 L_5 \\ + b^2 \tau_1 L_3 - \tau_1^2 x_1 R_1 (1+M_1)(1-M_2) \end{array}\right) - L_2 \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)\left(\frac{b\tau_1 L_5}{\in} + b\tau_1 L_3 L_5 + \frac{b^2\tau_1^2}{\in}\right) \\
& + \frac{bM_2 P_r \tau_1^2}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right) + \frac{L_1 \bar{j}_1 \tau_1}{\in}\left(\frac{bM_2 P_r \tau_1}{\in} + bM_2 P_r L_3 \tau_1\right) + \frac{\bar{j}_1^2 \tau_1}{\in}\left(\begin{array}{l} bM_2 P_r L_3 + bM_2 P_r \tau_1 L_4 \\ - x_1 R_1 M_1 \tau_1^2 (1-M_2) \end{array}\right) \\
& + \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)\left(\frac{bM_2 P_r \tau_1}{\in} + bM_2 P_r L_3 \tau_1\right)
\end{aligned}
$$

$$
\begin{aligned}
A_3 = & \frac{L_1 L_2 b \tau_1^2 L_5}{\in}\left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right) + \frac{L_1 L_2 \bar{j}_1 \tau_1}{\in}\left(bL_3 L_5 + \frac{\tau_1 b^2}{\in} + b\tau_1 L_4 L_5 + b^2 \tau_1 L_3 - \tau_1^2 x_1 R_1 (1+M_1)(1-M_2)\right) \\
& + L_1 L_2 \left(\bar{j} L_3 + \frac{\tau_1 L_1}{\in}\right)\left(\frac{b\tau_1 L_5}{\in} + b\tau_1 L_3 L_5 + \frac{b^2\tau_1^2}{\in}\right) + L_2 \bar{j}_1 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right)\left(\frac{b\tau_1 L_5}{\in} + b\tau_1 L_3 L_5 + \frac{b^2\tau_1^2}{\in}\right) \\
& + \frac{L_2 \bar{j}_1^2 \tau_1}{\in}\left(\begin{array}{l} bL_4 L_5 + b^2 L_3 + b^2 \tau_1 L_4 \\ -2\tau_1 x_1 R_1 (1+M_1)(1-M_2) - \tau_1 x_1 R_1 h (1+M_1) \end{array}\right) + \frac{L_2 \bar{j}_1 b \tau_1^2 L_5}{\in}\left(L_1 L_4 - \frac{N_1^2 b}{\in}\right) \\
& + L_2 \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)\left(bL_3 L_5 + \frac{\tau_1 b^2}{\in} + b\tau_1 L_4 L_5 + b^2 \tau_1 L_3 - \tau_1^2 x_1 R_1 (1+M_1)(1-M_2)\right) \\
& - \frac{L_1 \bar{j}_1 \tau_1}{\in}\left(bM_2 P_r L_3 + bM_2 P_r \tau_1 L_4 - x_1 R_1 M_1 \tau_1^2 (1-M_2)\right) - \frac{L_1 b M_2 P_r \tau_1^2}{\in}\left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right) \\
& - L_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)\left(\frac{bM_2 P_r \tau_1}{\in} + bM_2 P_r L_3 \tau_1\right) - \bar{j}_1 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in}\right)\left(\frac{bM_2 P_r \tau_1}{\in} + bM_2 P_r L_3 \tau_1\right) \\
& - \frac{\bar{j}_1^2 \tau_1}{\in}\left(bM_2 P_r L_4 - 2\tau_1 x_1 R_1 M_1 (1-M_1) - \tau_1 x_1 R_1 M_1 h\right) - \frac{\bar{j}_1 b M_2 P_r \tau_1^2}{\in}\left(L_1 L_4 - \frac{N_1^2 b}{\in}\right) \\
& - \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right)\left(bM_2 P_r L_3 + bM_2 P_r \tau_1 L_4 - x_1 R_1 M_1 \tau_1^2 (1-M_2)\right) \\
& - \frac{L_2 N_1 b \bar{j}_1 \tau_1}{\in^2}\left(2\tau_1 N_1 b L_5 + \tau_1^2 N_1 b - \tau_1^2 x_1 R_1 N_3' (1+M_1)\right) - \frac{L_2 N_1^2 b^2 L_5 \tau_1^2}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right) \\
& - \frac{N_1 b \bar{j}_1 \tau_1}{\in^2}\left(x_1 R_1 M_1 N_3' \tau_1^2 - 2\tau_1 M_2 P_r N_1 b\right) + \frac{M_2 P_r N_1^2 b^2 \tau_1^2}{\in}\left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in}\right) \\
& - \frac{T_{A_1}}{\in^2}\left(2\tau_1 \bar{j}_1 + 2\bar{j}_1 L_1 \tau_1^2\right)\left(M_2 P_r \tau_1 - L_2 L_5 \tau_1\right) + \frac{T_{A_1}}{\in^2}\tau_1^2 \bar{j}_1 \left(L_2 L_5 + L_2 \tau_1 b - M_2 P_r\right)
\end{aligned}
$$

$$A_4 = \bar{j}_1 L_2 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) \left(b L_3 L_5 + \frac{\tau_1 b^2}{\in} + b \tau_1 L_4 L_5 + b^2 \tau_1 L_3 - \tau_1^2 x_1 R_1 (1 + M_1)(1 - M_2) \right)$$

$$- \frac{L_2 \bar{j}_1^2 \tau_1}{\in} \left(x_1 R_1 (1 + M_1)(1 - M_2) + x_1 R_1 h (1 + M_1) - b^2 L_4 \right) + L_2 \bar{j}_1 \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right) \left(\frac{b \tau_1 L_5}{\in} + b \tau_1 L_3 L_5 + \frac{b^2 \tau_1^2}{\in} \right)$$

$$+ L_2 \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(b L_4 L_5 + b^2 L_3 + b^2 \tau_1 L_4 - 2 \tau_1 x_1 R_1 (1 + M_1)(1 - M_2) - \tau_1 x_1 R_1 h (1 + M_1) \right)$$

$$+ L_1 L_2 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) \left(\frac{b \tau_1 L_5}{\in} + b \tau_1 L_3 L_5 + \frac{b^2 \tau_1^2}{\in} \right) + \frac{L_1 L_2 \bar{j}_1 \tau_1}{\in} \left(\begin{array}{l} b L_4 L_5 + b^2 L_3 + b^2 \tau_1 L_4 \\ -2 \tau_1 x_1 R_1 (1 + M_1)(1 - M_2) - \tau_1 x_1 R_1 h (1 + M_1) \end{array} \right)$$

$$+ \frac{L_1 L_2 b \tau_1^2 L_5}{\in} \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right) + L_1 L_2 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(\begin{array}{l} b L_3 L_5 + \frac{\tau_1 b^2}{\in} + b \tau_1 L_4 L_5 + b^2 \tau_1 L_3 \\ - \tau_1^2 x_1 R_1 (1 + M_1)(1 - M_2) \end{array} \right)$$

$$- \bar{j}_1 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) \left(b M_2 P_r L_3 + b M_2 P_r \tau_1 L_4 - x_1 R_1 M_1 \tau_1^2 (1 - M_2) \right)$$

$$+ \frac{\bar{j}_1^2 \tau_1}{\in} \left(x_1 R_1 M_1 h + x_1 R_1 M_1 (1 - M_2) \right) - \bar{j}_1 \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right) \left(\frac{b M_2 P_r \tau_1}{\in} + b M_2 P_r L_3 \tau_1 \right)$$

$$- \bar{j}_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(b M_2 P_r L_4 - 2 \tau_1 x_1 R_1 M_1 (1 - M_2) - \tau_1 x_1 R_1 M_1 h \right) - L_1 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right)$$

$$\times \left(\frac{b M_2 P_r \tau_1}{\in} + b M_2 P_r L_3 \tau_1 \right) - \frac{L_1 \bar{j}_1 \tau_1}{\in} \left(b M_2 P_r L_4 - 2 \tau_1 x_1 R_1 M_1 (1 - M_2) - \tau_1 x_1 R_1 M_1 h \right)$$

$$- L_1 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(\begin{array}{l} b M_2 P_r L_3 + b M_2 P_r \tau_1 L_4 \\ - x_1 R_1 M_1 \tau_1^2 (1 - M_2) \end{array} \right) + \frac{L_2 N_1 b \bar{j} \tau_1}{\in^2} \left(\begin{array}{l} 2 \tau_1 x_1 R_1 N_3' (1 + M_1) \\ - N_1 b L_5 - 2 \tau_1 N_1 b^2 \end{array} \right)$$

$$- \frac{L_2 N_1^2 b^2 L_5 \tau_1^2}{\in} \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) - L_2 N_1 b \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(\begin{array}{l} 2 \tau_1 N_1 b L_5 + \tau_1^2 N_1 b^2 \\ - \tau_1^2 x_1 R_1 N_3' (1 + M_1) \end{array} \right)$$

$$+ \frac{\bar{j}_1 \tau_1 M_2 P_r N_1^2 b^2}{\in^2} - \frac{2 N_1 b \tau_1^2 \bar{j}_1 x_1 R_1 M_1 N_3'}{\in^2} + \frac{\bar{j}_1 L_4 M_2 P_r N_1^2 b^2 \tau_1^2}{\in} + \frac{L_1 L_3 M_2 P_r N_1^2 b^2 \tau_1^2}{\in} - \frac{L_1 b M_2 P_r \tau_1^2}{\in} \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right)$$

$$- \frac{\tau_1^3 N_1^4 b^3 M_2 P_r}{\in^2} - \frac{N_1 b}{\in} \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(x_1 R_1 M_1 N_3' \tau_1^2 - 2 \tau_1 P_r N_1 b \right)$$

$$+ \frac{T_{A_1}}{\in^2} \left(L_2 L_5 + L_2 \tau_1 b - M_2 P_r \right) \left(2 \tau_1 \bar{j}_1 + 2 \bar{j}_1 L_1 \tau_1^2 \right) + \frac{T_{A_1} L_2 b \tau_1^2 \bar{j}_1}{\in^2} - \frac{T_{A_1}}{\in^2} \left(4 \tau_1 \bar{j}_1 L_1 + \tau_1^2 L_1^2 + \bar{j}_1 \right) \left(M_2 P_r \tau_1 - L_2 L_5 \tau_1 \right)$$

$$A_5 = - L_1 L_2 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) \left(b L_3 L_5 + \frac{\tau_1 b^2}{\in} + b \tau_1 L_4 L_5 + b^2 \tau_1 L_3 - \tau_1^2 x_1 R_1 (1 + M_1)(1 - M_2) \right)$$

$$+ \frac{L_1 L_2 \bar{j}_1 \tau_1}{\in} \left(\begin{array}{l} x_1 R_1 (1 + M_1)(1 - M_2) \\ + x_1 R_1 h (1 + M_1) - b^2 L_4 \end{array} \right) - L_1 L_2 \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right) \left(\frac{b \tau_1 L_5}{\in} + b \tau_1 L_3 L_5 + \frac{b^2 \tau_1^2}{\in} \right)$$

$$- L_1 L_2 \left(\bar{j}_1 L_3 + \frac{\tau_1 L_1}{\in} \right) \left(b L_4 L_5 + b^2 L_3 + b^2 \tau_1 L_4 - 2 \tau_1 x_1 R_1 (1 + M_1)(1 - M_2) - \tau_1 x_1 R_1 h (1 + M_1) \right)$$

$$- L_2 \bar{j}_1 \left(\bar{j}_1 L_4 + L_1 L_3 - \frac{\tau_1 N_1^2 b}{\in} \right) \left(b L_4 L_5 + b^2 L_3 + b^2 \tau_1 L_4 - 2 \tau_1 x_1 R_1 (1 + M_1)(1 - M_2) - \tau_1 x_1 R_1 h (1 + M_1) \right)$$

$$-L_2\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(bL_3L_5+\frac{\tau_1b^2}{\in}+b\tau_1L_4L_5+b^2\tau_1L_3-\tau_1^2x_1R_1(1+M_1)(1-M_2)\right)$$

$$+L_2\bar{j}_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$+L_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(bM_2P_rL_3+bM_2P_r\tau_1L_4-x_1R_1M_1\tau_1^2(1-M_2)\right)$$

$$-\frac{L_1\bar{j}_1\tau_1}{\in}\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+L_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(\frac{bM_2P_r\tau_1}{\in}+bM_2P_rL_3\tau_1\right)$$

$$+L_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(bM_2P_rL_4-2\tau_1x_1R_1M_1(1-M_2)-\tau_1x_1R_1M_1h\right)$$

$$+\bar{j}_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\begin{pmatrix}bM_2P_rL_4-2\tau_1x_1R_1M_1(1-M_2)\\-\tau_1x_1R_1M_1h)\end{pmatrix}-\bar{j}_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\begin{pmatrix}x_1R_1M_1h\\+x_1R_1M_1(1-M_2))\end{pmatrix}$$

$$+\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(bM_2P_rL_3+bM_2P_r\tau_1L_4-x_1R_1M_1\tau_1^2(1-M_2)\right)+\frac{L_2N_1b}{\in}\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)$$

$$\times\left(2\tau_1N_1bL_5+\tau_1^2N_1b^2-\tau_1^2x_1R_1N_3'(1+M_1)-\frac{\bar{j}_1\tau_1}{\in}x_1R_1N_3'(1+M_1)\right)+\frac{\bar{j}_1\tau_1N_1b^2}{\in}\Bigg)+\frac{L_2N_1^2b^2L_5\tau_1^2}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)$$

$$-\frac{L_2N_1b}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(2\tau_1x_1R_1N_3'(1+M_1)-N_1bL_5-2\tau_1N_1b^2\right)+\frac{N_1b\bar{j}_1\tau_1}{\in^2}x_1R_1M_1N_3'$$

$$+\frac{N_1b}{\in}\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1M_1N_3'\tau_1^2-2\tau_1M_2P_rN_1b\right)-\frac{M_2P_rN_1^2b^2\tau_1^2}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)$$

$$-\frac{N_1b}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(M_2P_rN_1b-2\tau_1x_1R_1M_1N_3'\right)+\frac{T_{A_1}}{\in^2}\left(2\tau_1L_1^2+2\bar{j}_1L_1\right)\left(M_2P_r\tau_1-L_2L_5\tau_1\right)$$

$$-\frac{T_{A_1}}{\in^2}L_2b\left(2\tau_1\bar{j}_1+2\bar{j}_1L_1\tau_1^2\right)+\frac{T_{A_1}}{\in^2}\left(M_2P_r-L_2L_5-L_2\tau_1b\right)\left(4\tau_1\bar{j}_1L_1+L_1^2\tau_1^2+\bar{j}_1\right)$$

$$A_6=L_2\bar{j}_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$+L_2\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1)-bL_4L_5-b^2L_3-b^2\tau_1L_4\right)$$

$$+L_1L_2\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1)-bL_4L_5-b^2L_3-b^2\tau_1L_4\right)$$

$$-L_1L_2\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(bL_3L_5+\frac{\tau_1b^2}{\in}+b\tau_1L_4L_5+b^2\tau_1L_3-\tau_1^2x_1R_1(1+M_1)(1-M_2)\right)$$

$$+L_1L_2\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$-\bar{j}_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\begin{pmatrix}bM_2P_rL_4-2\tau_1x_1R_1M_1(1-M_2)\\-\tau_1x_1R_1M_1h\end{pmatrix}$$

$$+L_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(bM_2P_rL_4-2\tau_1x_1R_1M_1(1-M_2)-\tau_1x_1R_1M_1h\right)$$

$$-L_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+L_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\begin{pmatrix}bM_2P_rL_3+bM_2P_r\tau_1L_4\\-x_1R_1M_1\tau_1^2(1-M_2)\end{pmatrix}$$

$$-\frac{L_2N_1b}{\in}\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(2\tau_1x_1R_1N_3'(1+M_1)-N_1bL_5-2\tau_1N_1b^2\right)$$

$$+\frac{L_2N_1b}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(2\tau_1N_1bL_5+\tau_1^2N_1b^2-\tau_1^2x_1R_1N_3'(1+M_1)\right)$$

$$-\frac{L_2N_1b}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1N_3'(1+M_1)-N_1b^2\right)-\frac{N_1b}{\in}\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(M_2P_rN_1b-2\tau_1x_1R_1M_1N_3'\right)$$

$$+\frac{N_1b}{\in}x_1R_1M_1N_3'\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)+\frac{N_1b}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(x_1R_1M_1N_3'\tau_1^2-2\tau_1P_rN_1b\right)$$

$$+\frac{T_{A_1}}{\in^2}\left(2\tau_1L_1^2+2\bar{j}_1L_1\right)\left(M_2P_r-L_2L_5-L_2\tau_1b\right)-\frac{T_{A_1}}{\in^2}L_2b\left(4\tau_1\bar{j}_1L_1+\tau_1^2L_1^2+\bar{j}_1\right)+\frac{T_{A_1}}{\in^2}L_1^2\left(M_2P_r\tau_1-L_2L_5\tau_1\right)$$

$$A_7=-L_1L_2\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$+L_1L_2\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(bL_4L_5+b^2L_3+b^2\tau_1L_4-2\tau_1x_1R_1(1+M_1)(1-M_2)-\tau_1x_1R_1h(1+M_1)\right)$$

$$-L_2\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$+L_1\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+L_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\begin{pmatrix}2\tau_1x_1R_1M_1(1-M_2)\\+\tau_1x_1R_1M_1h-bM_2P_rL_4\end{pmatrix}$$

$$+\bar{j}_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\begin{pmatrix}x_1R_1M_1h\\+x_1R_1M_1(1-M_2)\end{pmatrix}+\frac{L_2N_1b}{\in}\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)\left(x_1R_1N_3'(1+M_1-N_1b^2\right)$$

$$+\frac{L_2N_1b}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(2\tau_1x_1R_1N_3'(1+M_1)-N_1bL_5-2\tau_1N_1b^2\right)-\frac{N_1b}{\in}x_1R_1M_1N_3'\left(\bar{j}_1L_4+L_1L_3-\frac{\tau_1N_1^2b}{\in}\right)$$

$$+\frac{N_1b}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(M_2P_rN_1b-2\tau_1x_1R_1M_1N_3'\right)+\frac{T_{A_1}}{\in^2}L_1^2\left(L_2L_5+\tau_1L_2b-M_2P_r\right)+\frac{T_{A_1}}{\in^2}L_2b\left(2\tau_1L_1^2+2\bar{j}_1L_1\right)$$

$$A_8=-L_1L_2\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)-b^2L_4\right)$$

$$+L_1\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+\frac{L_2N_1b}{\in}\left(L_1L_4-\frac{N_1^2b}{\in}\right)\left(x_1R_1N_3'(1+M_1)-N_1b^2\right)$$

$$-\frac{N_1b}{\in}x_1R_1M_1N_3'\left(L_1L_4-\frac{N_1^2b}{\in}\right)+\frac{T_{A_1}}{\in^2}L_1^2L_2b$$

VIII. STATIONARY CONVECTION

Here we consider the stationary convection in case $M_2\cong 0$, therefore in stationary convection, the marginal state will be characterized by putting $\sigma_1=0$ in equation (62) which reduces to

$$R_1 = \frac{L_2 b^2 \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right)^2 + \frac{T_{A_1}}{\in^2} L_1^2 L_2 b}{x_1 \left(L_1 L_4 - \frac{N_1^2 b}{\in} \right) \left[L_2 (1 + M_1) - M_1 \right] \left[L_1 h_1 - \frac{N_1 b N_3'}{\in} \right]}$$

where $h_1 = 1 + h$

On putting the values of L_1, L_2 and L_4, we obtain

$$R_1 = \frac{b^2 (1 + x_1 M_3) \left[\left(\frac{1 + N_1}{K_1'} \right) (2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2 + b (1 + x_1 M_3) \frac{T_{A_1}}{\in^2} \left[2N_1 + bN_2' \right]^2}{x_1 \left[\left(\frac{1 + N_1}{K_1'} \right) (2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right] \left[1 + x_1 M_3 (1 + M_1) \right] \left[h_1 (2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]} \quad ...(64)$$

In the absence of rotation (i.e., $T_{A_1} = 0$), equation (64) reduces to

$$R_1 = \frac{b^2 (1 + x_1 M_3) \left[\left(\frac{1 + N_1}{K_1'} \right) (2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]}{x_1 \left[1 + x_1 M_3 (1 + M_1) \right] \left[h_1 (2N_1 + bN_2') - \frac{N_1 N_3' b}{\in} \right]} \quad ...(65)$$

Which is similar to the equation derived by **R. Mittal** and **U.S. Rana[9].**

In case of ferromagnetic fluid, we set $N_1 = 0$ and keeping N_2' arbitrary in (65), we get

$$R_1 = \frac{b^2 (1 + x_1 M_3)}{x_1 h_1 K_1' \left[1 + x_1 M_3 (1 + M_1) \right]} = \frac{(1 + x_1)^2 (1 + x_1 M_3)}{x_1 h_1 K_1' \left[1 + x_1 M_3 (1 + M_1) \right]} \quad ...(66)$$

Which is the expression for Rayleigh number for ferromagnetic dusty fluid in a porous medium.

Further if we set $M_3 = 0$ in equation (66), we get

$$R_1 = \frac{(1 + x_1)^2}{x_1 h_1 K_1'} \quad ...(67)$$

which is the expression for the classical Rayleigh-Bénard number for Newtonian dusty fluid in porous medium.

Now to investigate the effects of medium permeability parameter (K_1') non-buoyancy magnetization parameter (M_3), micropolar coupling parameter (N_1), spin-diffusion parameter (N_2'), micropolar heat conduction parameter (N_3'), dust particles parameter (h_1) and rotation parameter T_{A_1}, we examine the behaviour of $\frac{dR_1}{dK_1'}, \frac{dR_1}{dM_3}, \frac{dR_1}{dN_1}, \frac{dR_1}{dN_2'}, \frac{dR_1}{dN_3'}, \frac{dR_1}{dh_1}$ and $\frac{dR_1}{dT_{A_1}}$ analytically as follows

From equation (64), we have

$$\frac{dR_1}{dK_1'} = \frac{-b (1 + x_1 M_3) \left(\frac{1 + N_1}{K_1'^2} \right) (2N_1 + bN_2')}{x_1 \left[1 + x_1 M_3 (1 + M_1) \right] \left[h_1 (2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]}$$

$$\times \left[\frac{(2N_1 + bN_2')^2 \left[b \left(\frac{1 + N_1}{K_1'} \right)^2 - \frac{T_{A_1}}{\in^2} \right] + \frac{b^3 N_1^3}{\in} \left[\frac{N_1}{\in} - \frac{2N_2'}{K_1'} \right] - \frac{4 N_1^3 b^2 (1 + N_1)}{\in K_1'} - \frac{2 N_1^2 b^3 N_2'}{\in K_1'}}{\left[\left(\frac{1 + N_1}{K_1'} \right) (2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2} \right]$$

If $T_{A_1} > \frac{N_1 \in^2}{N_2'} \left(\frac{1 + N_1}{K_1'} \right)^2$, $\max . \left\{ \frac{1}{2} \left(1 + \frac{1}{N_1} \right)^2, \frac{1}{N_1} \left(1 + \frac{1}{N_1} \right) \right\} < \frac{K_1'}{\in} < \frac{2N_2'}{N_1}$ and $h_1 N_2' > \frac{N_1 N_3'}{\in}$, then $\frac{dR_1}{dK_1'} > 0$, thus, under these conditions the medium permeability has stabilizing effect.

In the absence of rotation (i.e., $T_{A_1} = 0$), $\frac{dR_1}{dK_1'} < 0$ when $\frac{2N_2'}{N_1} < \frac{K_1'}{\in} < \frac{N_2'}{N_1^2}$ and $h_1 N_2' > \frac{N_1 N_3'}{\in}$, thus, under these conditions, the medium permeability has destabilizing effect in the absence of rotation.

Again from equation (64), we have

$$\frac{dR_1}{dM_3} = \frac{\left\{ b^2 \left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2 + b\frac{T_{A_1}}{\in^2}\left[2N_1 + bN_2' \right]^2 \right\}}{x_1 \left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]\left[h_1(2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]} \left[\frac{\left[1 + x_1 M_3(1+M_1)\right] x_1 - (1 + x_1 M_3)\left[x_1(1+M_1) \right]}{\left[1 + x_1 M_3 (1 + M_1) \right]^2} \right]$$

Clearly $\frac{dR_1}{dM_3} < 0$ when $\frac{1}{K_1'} > \frac{b}{2\in}$ and $N_2' > \frac{N_1 N_3'}{h_1 \in}$, thus the non-buoyancy magnetization has destabilizing effect under the conditions $\frac{1}{K_1'} > \frac{b}{2\in}$ and $N_2' > \frac{N_1 N_3'}{h_1 \in}$. In the absence of rotation, $\frac{dR_1}{dM_3} < 0$, under the same conditions, which implies that the destabilizing behaviour of non-buoyancy magnetization remains unaffected by rotation.

In the absence of rotation and micropolar viscous effects $(N_1 = 0)$, $\frac{dR_1}{dM_3}$ is always negative, implying the destabiizing effect of non-buyancy magnetization, which is derived by **Sunil** *et al.*[19].

Again from (64), we have

$$\frac{dR_1}{dN_1} = \frac{b(1 + x_1 M_3)}{x_1\left[1 + x_1 M_3(1+M_1)\right]} \times \left[\frac{\begin{array}{l} \left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]\left[h_1(2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]\left\{ 2b\left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right] \right. \\ \left. \times \left[\frac{2N_1 + bN_2'}{K_1'} + \frac{2(1+N_1)}{K_1'} - \frac{2N_1 b}{\in} \right] + 4\frac{T_{A_1}}{\in^2}(2N_1 + bN_2') \right\} - \left\{ b\left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2 + \frac{T_{A_1}}{\in^2}(2N_1 + bN_2')^2 \right\} \\ \times \left\{ \left[\frac{2N_1 + bN_2'}{K_1'} + \frac{2(1+N_1)}{K_1'} - \frac{2N_1 b}{\in} \right]\left[h_1(2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right] + \left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]\left[2h_1 - \frac{bN_3'}{\in} \right] \right\} \end{array}}{\left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2 \left[h_1(2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]^2} \right]$$

If $\frac{2}{K_1'} > \frac{b}{\in}$ or $\frac{1}{K_1'} > \frac{b}{2\in}$, $2h_1 - \frac{bN_3'}{\in} < 0$ or $h_1 < \frac{bN_3'}{2\in}$, $h_1 > \frac{N_3' N_1}{N_2' \in}$ and $\frac{\sqrt{b}}{K_1'} > \frac{\sqrt{T_{A_1}}}{\in}$ or $T_{A_1} < \frac{b\in^2}{K_1'^2}$, then $\frac{dR_1}{dN_1} > 0$, thus the coupling parameter has a stabilizing effect under the conditions

$$\frac{1}{K_1'} > \frac{b}{2\in}, \frac{N_3' N_1}{N_2' \in} < h_1 < \frac{bN_3'}{2\in} \text{ and } T_{A_1} < \frac{b\in^2}{K_1'^2}$$

In the absence of rotation, $\frac{dR_1}{dN_1}$ will always be positive if $\frac{1}{K_1'} > \frac{b}{2\in}$ and $h_1 > \frac{bN_3'}{\in}$, which is derived by **R. Mittal** *et. al*[9].

Again from (64), we have

$$\frac{dR_1}{dN_2'} = \frac{-b\left(1+x_1M_3\right)}{x_1\left[1+x_1M_3(1+M_1)\right]} \times \left[\frac{\begin{array}{l} b^2\left[\left(\frac{1+N_1}{K_1'}\right)(2N_1+bN_2') - \frac{N_1^2 b}{\in}\right]^2 \left[\frac{N_3'}{K_1'} + N_1\left(\frac{N_3'}{K_1'} - h_1\right)\right] + \frac{b^2 N_1 N_3'}{\in^2} T_{A_1}(2N_1+bN_2') \\ \times\left[\frac{2N_1+bN_2'}{K_1'} + \frac{bN_1N_3'}{K_1'} + N_1^2\left(\frac{2}{K_1'} - \frac{b}{\in}\right)\right] + \frac{b^2 N_1^2}{\in^3} T_{A_1}(2N_1+bN_2')\left[2h_1N_1 + b\left(h_1N_2' - \frac{N_1N_3'}{\in}\right)\right] \end{array}}{\left[\left(\frac{1+N_1}{K_1'}\right)(2N_1+bN_2') - \frac{N_1^2 b}{\in}\right]^2 \left[h_1(2N_1+bN_2') - \frac{N_1bN_3'}{\in}\right]^2}\right]$$

If $\frac{N_3'}{K_1'} - h_1 > 0$ or $h_1 < \frac{N_3'}{K_1'}$, $\frac{2}{K_1'} - \frac{b}{\in} > 0$ or $\frac{1}{K_1'} > \frac{b}{2\in}$ and $h_1N_2' - \frac{N_1N_3'}{\in} > 0$ or $h_1 > \frac{N_1N_3'}{N_2'\in}$, then $\frac{dR_1}{dN_2'} < 0$, which implies that the spin-diffusion parameter has a destabilizing effect when

$$\frac{N_1N_3'}{N_2'\in} < h_1 < \frac{N_3'}{K_1'} \text{ and } \frac{1}{K_1'} > \frac{b}{2\in}$$

In the absence of rotation, $\frac{dR_1}{dN_2'}$ is always negative, when $h_1 < \frac{N_3'}{K_1'}$ which is derived by **R. Mittal** *et. al*[**9**]. Thus the presence of rotation affects the dust particles i.e., it gives also the lower bound of h_1 but in the absence of rotation, only upper bound of h_1 was obtained.

From (64), we have

$$\frac{dR_1}{dN_3'} = \frac{N_1 b}{\in x_1}\left[\frac{b^2(1+x_1M_3)\left[\left(\frac{1+N_1}{K_1'}\right)(2N_1+bN_2') - \frac{N_1^2 b}{\in}\right]^2 + b(1+x_1M_3)\frac{T_{A_1}}{\in^2}(2N_1+bN_2')^2}{\left[\frac{2N_1+bN_2'}{K_1'} + \frac{N_1bN_2'}{K_1'} + N_1^2\left(\frac{2}{K_1'} - \frac{b}{\in}\right)\right]\left[1+x_1M_3(1+M_1)\right]\left[h_1(2N_1+bN_2') - \frac{N_1bN_3'}{\in}\right]^2}\right]$$

If $\frac{2}{K_1'} - \frac{b}{\in} > 0$ or $\frac{1}{K_1'} > \frac{b}{2\in}$, then $\frac{dR_1}{dN_3'} > 0$, thus the micropolar heat conduction parameter has a stabilizing effect when $\frac{1}{K_1'} > \frac{b}{2\in}$, which is derived by **R. Mittal** *et. al*[**9**]. Hence the stabilizing behaviour of micropolar heat conduction parameter is independent of presence of rotation.

From (64), we have

$$\frac{dR_1}{dh_1} = \frac{-(2N_1+bN_2')}{x_1\left[1+x_1M_3(1+M_1)\right]}\left[\frac{b^2(1+x_1M_3)\left[\left(\frac{1+N_1}{K_1'}\right)(2N_1+bN_2') - \frac{N_1^2 b}{\in}\right]^2 + b(1+x_1M_3)\frac{T_{A_1}}{\in^2}(2N_1+bN_2')^2}{\left[\frac{2N_1+bN_2'}{K_1'} + \frac{N_1bN_2'}{K_1'} + N_1^2\left(\frac{2}{K_1'} - \frac{b}{\in}\right)\right]\left[h_1(2N_1+bN_2') - \frac{N_1bN_3'}{\in}\right]^2}\right]$$

If $\frac{2}{K_1'} - \frac{b}{\in} > 0$ or $\frac{1}{K_1'} > \frac{b}{2\in}$, then $\frac{dR_1}{dh_1} < 0$, thus the dust particles has a destabilizing effect when $\frac{1}{K_1'} > \frac{b}{2\in}$.
Hence the destabilizing behaviour of dust particles is independent of rotation.
From (64) we have

$$\frac{dR_1}{dT_{A_1}} = \frac{b(1+x_1M_3)(2N_1+bN_2')^2}{x_1\in^2\left[\frac{2N_1+bN_2'}{K_1'} + \frac{N_1bN_2'}{K_1'} + N_1^2\left(\frac{2}{K_1'} - \frac{b}{\in}\right)\right]\left[1+x_1M_3(1+M_1)\right]\left[2h_1N_1 + b\left(h_1N_2' - \frac{N_1N_3'}{\in}\right)\right]}$$

If $\frac{2}{K_1'} - \frac{b}{\in} > 0$ or $\frac{1}{K_1'} > \frac{b}{2\in}$ and $h_1 N_2' > \frac{N_1 N_3'}{\in}$, then $\frac{dR_1}{dT_{A_1}} > 0$, thus the rotation has a stabilizing effect when $\frac{1}{K_1'} > \frac{b}{2\in}$ and $h_1 N_2' > \frac{N_1 N_3'}{\in}$. In the absence of micropolar viscous effect $(N_1 = 0)$, $\frac{dR_1}{dT_{A_1}}$ is always positive which is derived by **Sunil** *et. al***[19].**

For sufficiently large value of M_1, we obtain the result for magnetic mechanism as follows:

$$R_m = \frac{1 + x_1 M_3}{x_1^2 M_3} \left[\frac{b^2 \left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right]^2 + \frac{bT_{A_1}}{\in^2}(2N_1 + bN_2')^2}{\left[\left(\frac{1+N_1}{K_1'} \right)(2N_1 + bN_2') - \frac{N_1^2 b}{\in} \right] \left[h_1 (2N_1 + bN_2') - \frac{N_1 b N_3'}{\in} \right]} \right] \quad \left[\because M_1 \text{ is large so that } \frac{1}{M_1} \to 0 \right]$$

Where R_m is the magnetic thermal Rayleigh number.

$$R_m = \frac{(1 + x_1 M_3)\left[(1+x_1)^4 S_1 + (1+x_1)^3 S_2 + (1+x_1)^2 S_3 + (1+x_1) S_4 \right]}{x_1^2 M_3 \left[(1+x_1)^2 S_5 + (1+x_1) S_6 + S_7 \right]} \qquad ...(68)$$

Where $S_1 = \left(\frac{N_2' + N_2' N_1}{K_1'} - \frac{N_1^2}{\in} \right)^2$, $S_2 = 2\left(\frac{2N_1 + 2N_1^2}{K_1'} \right)\left(\frac{N_2' + N_2' N_1}{K_1'} - \frac{N_1^2}{\in} \right) + \frac{T_{A_1} N_2'^2}{\in^2}$,

$S_3 = \left(\frac{2N_1 + 2N_1^2}{K_1'} \right)^2 + \frac{4 N_1 N_2' T_{A_1}}{\in^2}$, $S_4 = \frac{4 T_{A_1} N_1^2}{\in^2}$, $S_5 = \left(\frac{N_2' + N_2' N_1}{K_1'} - \frac{N_1^2}{\in} \right)\left(h_1 N_2' - \frac{N_1 N_3'}{\in} \right)$,

$S_6 = \left(\frac{2N_1 + 2N_1^2}{K_1'} \right)\left(h_1 N_2' - \frac{N_1 N_3'}{\in} \right) + 2h_1 N_1 \left(\frac{N_2' + N_2' N_1}{K_1'} - \frac{N_1^2}{\in} \right)$, $S_7 = 2h_1 N_1 \left(\frac{2N_1^2 + 2N_1}{K_1'} \right)$

If $N_1 = 0$, then equation (69) reduces to

$$R_m = \frac{(1 + x_1 M_3)\left[(1+x_1)^2 + \frac{T_{A_1} K_1'^2}{\in^2}(1+x_1) \right]}{x_1^2 M_3 h_1 K_1'}$$

Which is similar to expression derived by **Sunil** *et. al***[19].** For very large value of M_3, equation (68) reduces to

$$R_m = \frac{\left[(1+x_1)^4 S_1 + (1+x_1)^3 S_2 + (1+x_1)^2 S_3 + (1+x_1) S_4 \right]}{x_1 \left[(1+x_1)^2 S_5 + (1+x_1) S_6 + S_7 \right]} = R_1 \text{ (in the absence of magnetic parameter)}$$

Thus, in case of stationary convection, the ferromagnetic micropolar fluid behaves like an ordinary micropolar fluid for sufficiently large values of M_1 and M_3.

Now differentiating w.r.t. to x_1 on both sides of (68), we have

$$\frac{dR_m}{dx_1} = \left[\frac{\begin{array}{l} -x_1 M_3 (2 + x_1 M_3)\left[(1+x_1)^6 S_1 S_5 + (1+x_1)^5 \{ S_2 S_5 + S_1 S_6 \} + (1+x_1)^4 \{ S_3 S_5 + S_2 S_6 + S_1 S_7 \} \right. \\ \left. + (1+x_1)^3 \{ S_4 S_5 + S_3 S_6 + S_2 S_7 \} + (1+x_1)^2 \{ S_4 S_6 + S_3 S_7 \} + (1+x_1)\{ S_4 S_7 \} \right] \\ + x_1^2 M_3 (1 + x_1 M_3)\left[(1+x_1)^5 (2 S_1 S_5) + (1+x_1)^4 \{ S_2 S_5 + 3 S_1 S_6 \} + (1+x_1)^3 \{ 2 S_2 S_6 + 4 S_1 S_7 \} \right. \\ \left. + (1+x_1)^2 \{ S_3 S_6 + 3 S_2 S_7 - S_4 S_5 \} + (1+x_1)\{ 2 S_3 S_7 \} + S_7 S_4 \right] \end{array}}{x_1^4 M_3^2 \left[(1+x_1)^2 S_5 + (1+x_1) S_6 + S_7 \right]^2} \right]$$

For minimum value of R_m we must have $\frac{dR_m}{dx_1} = 0$

$$x_1^7 \left(M_3 S_5^2 - M_3 S_1 S_5 \right) + x_1^6 M_3 \left\{ 4S_5^2 + 2S_5 S_6 - 4S_1 S_5 - 2S_1 S_6 \right\}$$

$$+ x_1^5 \left\{ \begin{matrix} M_3 \left(6S_5^2 + S_6^2 + 6S_5 S_6 + 2S_5 S_7 - 5S_1 S_5 - S_2 S_5 - 7S_1 S_6 + 5S_3 S_5 - S_2 S_6 - 3S_1 S_7 \right) \\ +2S_1 S_5 + S_2 S_5 - S_1 S_6 \end{matrix} \right\}$$

$$+ x_1^4 \left\{ M_3 \left(\begin{matrix} 4S_5^2 + 2S_6^2 + 6S_5 S_6 + 4S_5 S_7 + 2S_6 S_7 + 4S_2 S_5 - 8S_1 S_6 + 4S_3 S_5 - 2S_2 S_6 - 8S_1 S_7 + S_4 S_5 \\ -2S_2 S_7 + S_4 S_5 \end{matrix} \right) \right.$$

$$\left. +10S_1 S_5 + 6S_2 S_5 - 2S_1 S_6 + 2S_3 S_5 - 2S_1 S_7) \right\} + x_1^3 \left\{ M_3 \left(\begin{matrix} S_5^2 + S_6^2 + S_7^2 + 2S_5 S_6 + 2S_5 S_7 + 2S_6 S_7 \\ +5S_1 S_5 + 6S_2 S_5 - 2S_1 S_6 + 6S_3 S_5 - 6S_1 S_7 \\ +3S_4 S_5 + S_3 S_6 - 3S_2 S_7 + 3S_4 S_5 + S_4 S_6 - S_3 S_7 \end{matrix} \right) \right.$$

$$\left. +20S_1 S_5 + 14S_2 S_5 + 2S_1 S_6 + 8S_3 S_5 + 2S_2 S_6 - 4S_1 S_7 + 2S_4 S_5 + S_3 S_6 - S_2 S_7 + S_4 S_5 \right\}$$

$$x_1^2 \left\{ M_3 \left(4S_1 S_5 + 4S_2 S_5 + 2S_1 S_6 + 4S_3 S_5 + 2S_2 S_6 + 3S_4 S_5 + 2S_3 S_6 + 2S_4 S_6 + S_4 S_5 \right) \right.$$

$$\left. +20S_1 S_5 + 16S_2 S_5 + 8S_1 S_6 + 12S_3 S_5 + 6S_2 S_6 + 6S_4 S_5 + 4S_3 S_6 + 2S_4 S_6 + 3S_4 S_5 \right\}$$

$$+ x_1 \left\{ M_1 \left(S_1 S_5 + S_2 S_5 + S_1 S_6 + S_3 S_5 + S_2 S_6 + S_1 S_7 + S_4 S_5 + S_3 S_6 + S_2 S_7 + S_4 S_6 + S_3 S_7 + S_4 S_7 \right) \right.$$

$$\left. +10S_1 S_5 + 9S_2 S_5 + 7S_1 S_6 + 8S_3 S_5 + 6S_2 S_6 + 4S_1 S_7 + 7S_4 S_5 + 5S_3 S_6 + 3S_2 S_7 + 2S_3 S_7 + S_4 S_7 \right\}$$

$$+ \left\{ \begin{matrix} 2S_1 S_5 + 2S_2 S_5 + 2S_1 S_6 + 2S_3 S_5 + 2S_2 S_6 + 2S_1 S_7 + 2S_4 S_5 + 2S_3 S_6 \\ +2S_2 S_7 + 2S_4 S_6 + 2S_3 S_7 + 2S_4 S_7 \end{matrix} \right\} = 0 \qquad ...(69)$$

Hence the magnetic thermal number R_m has its minimum value when condition (69) holds.

IX. OSCILLATORY CONVECTION

Comparing the imaginary parts of both sides of equation (63), we get

$$\sigma_1 \left[A_1 \sigma_1^6 + A_3 \sigma_1^4 + A_5 \sigma_1^2 + A_7 \right] = 0 \qquad ...(70)$$

Where $A_1 = b^2 \left\{ -\frac{L_2 \bar{j}_1^2 \tau_1^3}{\epsilon^2} \right\} + b \left\{ -\frac{2L_1 L_2 \bar{j}_1 \tau_1^3 L_5}{\epsilon^2} - \frac{L_2 \bar{j}_1^2 \tau_1^2 L_5}{\epsilon^2} - \frac{2L_2 \bar{j}_1^2 \tau_1^2 L_3 L_5}{\epsilon} \right.$

$$\left. + \frac{\bar{j}_1^2 \tau_1^2 M_2 P_r}{\epsilon^2} + \frac{\bar{j}_1^2 \tau_1^2 M_2 P_r L_3}{\epsilon} + \frac{L_1 M_2 P_r \tau_1^2}{\epsilon} + \frac{\bar{j}_1^2 M_2 P_r \tau_1^2 L_3}{\epsilon} + \frac{\bar{j}_1 M_2 P_r \tau_1^3 L_1}{\epsilon^2} \right\}$$

$$A_3 = b^3 \left\{ -\frac{L_2 \bar{j}_1 \tau_1^3 N_1^2}{\epsilon^2} \right\} + b^2 \left\{ -\frac{L_1 L_2 \tau_1^3 L_5 N_1^2}{\epsilon^2} + \frac{L_1 L_2 \bar{j}_1 \tau_1^2}{\epsilon^2} + \frac{L_1 L_2 L_3 \bar{j}_1 \tau_1^2}{\epsilon} + \frac{L_1 L_2 L_3 \bar{j}_1 \tau_1^2}{\epsilon} + \frac{L_1^2 L_2 \tau_1^3}{\epsilon^2} \right.$$

$$+ \frac{L_2 L_4 \bar{j}_1^2 \tau_1^2}{\epsilon} + \frac{L_1 L_2 L_3 \bar{j}_1 \tau_1^2}{\epsilon} - \frac{\tau_1^2 N_1^2 L_5}{\epsilon^2} - \frac{\tau_1^2 N_1^2 L_3 L_5}{\epsilon} + \frac{L_2 L_3 \bar{j}_1^2 \tau_1}{\epsilon} + \frac{L_2 L_4 \bar{j}_1^2 \tau_1^2}{\epsilon} - \frac{L_2 L_5 \bar{j}_1 \tau_1^2 N_1^2}{\epsilon^2}$$

$$+ \frac{L_2 L_3 \bar{j}_1^2 \tau_1}{\epsilon} + \frac{L_1 L_2 \tau_1^2 \bar{j}_1}{\epsilon^2} + L_2 L_3^2 \bar{j}_1^2 \tau_1 + \frac{L_1 L_2 L_3 \bar{j}_1 \tau_1^2}{\epsilon} + \frac{L_1 M_2 P_r \tau_1^3 N_1^2}{\epsilon^2} + \frac{\bar{j}_1 \tau_1^2 N_1^2 M_2 P_r}{\epsilon^2} + \frac{\bar{j}_1 \tau_1^2 N_1^2 M_2 P_r L_3}{\epsilon}$$

$$+ \frac{\bar{j}_1 M_2 P_r \tau_1^2 N_1^2}{\epsilon^2} - \frac{2L_2 L_5 \tau_1^2 \bar{j}_1 N_1^2}{\epsilon^2} - \frac{L_2 \tau_1^3 N_1^2 \bar{j}_1}{\epsilon^2} - \frac{L_2 L_5 L_3 \bar{j}_1 N_1^2 \tau_1^2}{\epsilon} - \frac{L_1 L_2 L_5 \tau_1^3 N_1^2}{\epsilon^2} + \frac{2\tau_1^2 N_1^2 M_2 P_r \bar{j}_1}{\epsilon^2}$$

$$\left. + \frac{L_3 \bar{j}_1 M_2 P_r N_1^2 \tau_1^2}{\epsilon} + \frac{L_1 \tau_1^3 M_2 P_r N_1^2}{\epsilon^2} \right\} + b \left\{ \frac{L_1 L_2 L_4 \bar{j}_1 \tau_1^2 L_5}{\epsilon} + \frac{L_1^2 L_2 L_3 L_5 \tau_1^2}{\epsilon} + \frac{L_1 L_2 L_3 L_5 \bar{j}_1 \tau_1}{\epsilon} + \frac{L_1 L_2 L_4 L_5 \tau_1^2 \bar{j}_1}{\epsilon} \right.$$

$$+ \frac{L_1 L_2 L_3 L_5 \bar{j}_1 \tau_1}{\epsilon} + \frac{L_1^2 L_2 L_5 \tau_1^2}{\epsilon^2} + L_1 L_2 L_3^2 L_5 \bar{j}_1 \tau_1 + \frac{L_1^2 L_2 L_3 L_5 \tau_1^2}{\epsilon} + \frac{L_2 L_4 L_5 \bar{j}_1^2 \tau_1}{\epsilon} + \frac{L_1 L_2 L_3 L_4 \bar{j}_1 \tau_1}{\epsilon}$$

$$+ L_2 L_3 L_4 L_5 \bar{j}_1^2 \tau_1 + L_1 L_2 L_3^2 L_5 \bar{j}_1 \tau_1 + \frac{L_2 L_4 L_5 \bar{j}_1^2 \tau_1}{\epsilon} + \frac{L_1 L_2 L_4 L_5 \bar{j}_1 \tau_1^2}{\epsilon} + L_2 L_3^2 L_5 \bar{j}_1^2 + \frac{L_1 L_2 L_3 L_5 \bar{j}_1 \tau_1}{\epsilon}$$

$$+L_2L_3L_4L_5\bar{j}_1^2\tau_1+\frac{L_1L_2L_4L_5\bar{j}_1\tau_1^2}{\in}-\frac{L_1L_3M_2P_r\bar{j}_1\tau_1}{\in}-\frac{L_1L_4\tau_1^2\bar{j}_1M_2P_r}{\in}-\frac{L_1L_4\bar{j}_1M_2P_r\tau_1^2}{\in}-\frac{L_1^2L_3M_2P_r\tau_1^2}{\in}$$

$$-\frac{L_1L_3\bar{j}_1\tau_1M_2P_r}{\in}-\frac{L_1^2\tau_1^2M_2P_r}{\in^2}-L_1L_3^2\bar{j}_1\tau_1M_2P_r-\frac{L_1^2L_3\tau_1^2M_2P_r}{\in}-\frac{L_4\bar{j}_1^2\tau_1M_2P_r}{\in}-\frac{L_1L_3\bar{j}_1\tau_1M_2P_r}{\in}-L_3L_4\bar{j}_1^2\tau_1M_2P_r$$

$$-L_1L_3^2\bar{j}_1\tau_1M_2P_r-\frac{L_4\bar{j}_1^2\tau_1M_2P_r}{\in}-\frac{L_1L_4\bar{j}_1\tau_1M_2P_r}{\in}-L_3^2\bar{j}_1^2M_2P_r-\frac{L_1L_3\bar{j}_1\tau_1M_2P_r}{\in}-L_3L_4\bar{j}_1^2\tau_1M_2P_r$$

$$\left.-\frac{L_1L_4\bar{j}_1\tau_1^2M_2P_r}{\in}+\frac{L_2N_1\bar{j}_1\tau_1^3x_1R_1N_3'(1+M_1)}{\in^2}-\frac{N_1\bar{j}_1\tau_1^3x_1R_1M_1N_3'}{\in^2}+\frac{L_2\bar{j}_1\tau_1^3T_{A_1}}{\in^2}\right\}$$

$$+\left\{-\frac{L_1L_2\bar{j}_1}{\in}\tau_1^3x_1R_1(1+M_1)(1-M_2)-\frac{2L_2\bar{j}_1^2}{\in}\tau_1^2x_1R_1(1+M_1)(1-M_2)-\frac{L_2\bar{j}_1^2\tau_1^2x_1R_1h(1+M_1)}{\in}\right.$$

$$-L_2L_3\bar{j}_1^2\tau_1^2x_1R_1(1+M_1)(1-M_2)-\frac{L_1L_2\bar{j}_1\tau_1^3}{\in}x_1R_1(1+M_1)(1-M_2)+\frac{L_1\bar{j}_1\tau_1^3}{\in}x_1R_1M_1(1-M_2)$$

$$+\frac{2\bar{j}_1^2\tau_1^2}{\in}x_1R_1M_1(1-M_2)+\frac{\bar{j}_1^2\tau_1^2}{\in}x_1R_1M_1h+L_3\bar{j}_1^2\tau_1^2x_1R_1M_1(1-M_2)+\frac{L_1\bar{j}_1\tau_1^3}{\in}x_1R_1M_1(1-M_2)$$

$$\left.-\frac{T_{A_1}}{\in^2}\left(2\tau_1\bar{j}_1+2\bar{j}_1L_1\tau_1^2\right)\left(M_2P_r\tau_1-L_2L_5\tau_1\right)+\frac{L_2L_5\bar{j}_1\tau_1^2T_{A_1}}{\in^2}-\frac{\bar{j}_1\tau_1^2M_2P_rT_{A_1}}{\in^2}\right\}$$

$$A_5=b^4\left\{-\frac{L_2N_1^3\tau_1}{\in^2}\left(\tau_1^2N_1+\frac{\bar{j}_1\tau_1N_1}{\in}\right)\right\}+b^3\left\{\frac{L_1L_2\tau_1N_1^2}{\in}\left(\frac{\tau_1}{\in}+\tau_1L_3\right)+\frac{L_1L_2N_1^2\tau_1^2}{\in^2}\right.$$

$$+\frac{L_2\bar{j}_1\tau_1N_1^2}{\in}(L_3+\tau_1L_4)+\frac{L_2\bar{j}_1N_1^2}{\in}\left(\frac{\tau_1}{\in}+\tau_1L_3\right)+\frac{L_2N_1}{\in}\left(\bar{j}_1L_4+L_1L_3\right)\left(\tau_1^2N_1+\frac{\bar{j}_1\tau_1N_1}{\in}\right)$$

$$\left.-\frac{2L_2N_1^4\tau_1^2L_5}{\in^2}-\frac{L_2N_1^4L_5\tau_1^2}{\in^2}+\frac{2L_2N_1^2\tau_1}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)+\frac{2\tau_1^2N_1^4M_2P_r}{\in^2}+\frac{M_2P_rN_1^4\tau_1^4}{\in^2}\right\}$$

$$+b^2\left\{-L_1L_2\left(\bar{j}_1L_4+L_1L_3\right)\left(\frac{\tau_1}{\in}+\tau_1L_3\right)+\frac{L_1L_2\tau_1N_1^2}{\in}\left(L_3L_5+\tau_1L_4L_5\right)-\frac{L_1L_2L_4\bar{j}_1\tau_1}{\in}-\frac{L_1^2L_2L_4\tau_1^2}{\in}\right.$$

$$+\frac{L_1L_2N_1^2}{\in}\left(\frac{\tau_1L_5}{\in}+\tau_1L_3L_5\right)-L_1L_2\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(L_3+\tau_1L_4\right)-L_2\bar{j}_1\left(\bar{j}_1L_4+L_1L_3\right)\left(L_3+\tau_1L_4\right)$$

$$+\frac{L_2L_4L_5\bar{j}_1\tau_1N_1^2}{\in}-L_2\bar{j}_1L_1L_4\left(\frac{\tau_1}{\in}+\tau_1L_3\right)+\frac{L_2\bar{j}_1N_1^2}{\in}\left(L_3L_5+\tau_1L_4L_5\right)-L_2L_4\bar{j}_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)$$

$$-\frac{L_1N_1^2\tau_1}{\in}\left(M_2P_rL_3+M_2P_r\tau_1L_4\right)-\frac{L_1N_1^2}{\in}\left(\frac{M_2P_r\tau_1}{\in}+M_2P_rL_3\tau_1\right)-\frac{\bar{j}_1\tau_1N_1^2M_2P_rL_4}{\in}-\frac{\bar{j}_1N_1^2}{\in}\begin{pmatrix}M_2P_rL_3\\+M_2P_r\tau_1L_4\end{pmatrix}$$

$$+\frac{2\tau_1N_1^2L_2L_5}{\in}\left(\bar{j}_1L_4+L_1L_3\right)-\frac{L_2N_1^3\tau_1}{\in^2}\left(-\tau_1^2x_1R_1N_3'(1+M_1)-\frac{\tau_1\bar{j}_1}{\in}x_1R_1N_3'(1+M_1)\right)$$

$$+\frac{L_1L_2L_4L_5N_1^2\tau_1^2}{\in}+\frac{L_2N_1^2L_5}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)-\frac{N_1^3\tau_1^3}{\in^2}x_1R_1M_1N_3'-\frac{2N_1^2\tau_1M_2P_r}{\in}\left(\bar{j}_1L_4+L_1L_3\right)$$

$$\left.-\frac{L_1L_4M_2P_rN_1^2\tau_1^2}{\in}-\frac{N_1^2M_2P_r}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\right\}+b\left\{-L_1L_2\left(\bar{j}_1L_4+L_1L_3\right)\left(L_3L_5+\tau_1\tau_4L_5\right)\right.$$

$$+\frac{L_1L_2\tau_1^3N_1^2}{\in}x_1R_1(1+M_1)(1-M_2)-L_1^2L_2L_4\left(\frac{\tau_1L_5}{\in}+\tau_1L_3L_5\right)$$

$$-L_1L_2L_4L_5\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)-L_2L_4L_5\bar{j}_1\left(\bar{j}_1L_4+L_1L_3\right)$$

$$-\frac{L_2\bar{j}_1\tau_1N_1^2}{\in}\left(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1)\right)-L_1L_2L_4\bar{j}_1\left(L_3L_5+\tau_1L_4L_5\right)$$

$$+\frac{L_2\bar{j}_1N_1^2\tau_1^2}{\in}x_1R_1(1+M_1)(1-M_2)+L_1(\bar{j}_1L_4+L_1L_3)\left(M_2P_rL_3+M_2P_r\tau_1L_4\right)$$

$$-\frac{L_1\tau_1^3N_1^2}{\in}x_1R_1M_1(1-M_2)+L_1^2L_4\left(\frac{M_2P_r\tau_1}{\in}+M_2P_rL_3\tau_1\right)$$

$$+L_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)M_2P_rL_4+\bar{j}_1L_4M_2P_r(\bar{j}_1L_4+L_1L_3)$$

$$+\frac{\bar{j}_1\tau_1N_1^2}{\in}\left(2\tau_1x_1R_1M_1(1-M_2)+\tau_1x_1R_1M_1h\right)+\bar{j}_1L_1L_4\left(M_2P_rL_3+M_2P_r\tau_1L_4\right)$$

$$+\frac{\bar{j}_1N_1^2}{\in}x_1R_1M_1\tau_1^2(1-M_2)-\frac{L_2N_1}{\in}\left(\bar{j}_1L_4+L_1L_3\right)\left(\tau_1^2x_1R_1N_3'(1+M_1)\right)$$

$$+\frac{\bar{j}_1\tau_1}{\in}x_1R_1N_3'(1+M_1)-\frac{2L_2N_1\tau_1}{\in}x_1R_1N_3'(1+M_1)\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)$$

$$+\frac{N_1\bar{j}_1\tau_1}{\in^2}x_1R_1M_1N_3'+\frac{N_1}{\in}\left(\bar{j}_1L_4+L_1L_3\right)x_1R_1M_1N_3'\tau_1^2$$

$$+\frac{2N_1\tau_1}{\in}\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)x_1R_1M_1N_3'-\frac{T_{A_1}L_2}{\in^2}\left(2\bar{j}_1\tau_1+2\bar{j}_1L_1\tau_1^2\right)-\frac{L_2\tau_1T_{A_1}}{\in^2}\left(4\tau_1\bar{j}_1L_1+L_1^2\tau_1^2+\bar{j}_1\right)\Big\}$$

$$+\Big\{L_1L_2(\bar{j}_1L_4+L_1L_3)\tau_1^2x_1R_1(1+M_1)(1-M_2)+\frac{L_1L_2\bar{j}_1\tau_1}{\in}\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)\right)$$

$$+L_1L_2\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1))$$

$$+L_2\bar{j}_1(\bar{j}_1L_4+L_1L_3)\left(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1)\right)$$

$$+L_2\bar{j}_1L_1L_4\tau_1^2x_1R_1(1+M_1)(1-M_2)+L_2\bar{j}_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)\right)$$

$$-L_1(\bar{j}_1L_4+L_1L_3)x_1R_1M_1\tau_1^2(1-M_2)-\frac{L_1\bar{j}_1\tau_1}{\in}\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)$$

$$-L_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\begin{pmatrix}2\tau_1x_1R_1M_1(1-M_2)\\+\tau_1x_1R_1M_1h\end{pmatrix}-\bar{j}_1(\bar{j}_1L_4+L_1L_3)\left(2\tau_1x_1R_1M_1(1-M_2)+\tau_1x_1R_1M_1h\right)$$

$$-\bar{j}_1\left(\bar{j}_1L_3+\frac{\tau_1L_1}{\in}\right)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)-\bar{j}_1L_1L_4x_1R_1M_1\tau_1^2(1-M_2)$$

$$+\frac{T_{A_1}}{\in^2}\left(2\tau_1L_1^2+2\bar{j}_1L_1\right)\left(M_2P_r\tau_1-L_2L_5\tau_1\right)+\frac{T_{A_1}}{\in^2}(M_2P_r-L_2L_5)(4\tau_1\bar{j}_1L_1+L_1^2\tau_1^2+\bar{j}_1)\Big\}$$

$$A_7=b^4\left\{\frac{3L_2N_1^4\tau_1}{\in^2}\right\}+b^3\Big\{-\frac{L_1L_2L_4\tau_1N_1^2}{\in}-\frac{L_1L_2N_1^2}{\in}(L_3+\tau_1L_4)-\frac{L_2L_4\bar{j}_1N_1^2}{\in}$$

$$-\frac{L_2N_1^2}{\in}(\bar{j}_1L_4+L_1L_3)-\frac{N_1^4M_2P_r}{\in^2}-\frac{2L_1L_2L_4N_1^2\tau_1}{\in}+\frac{L_2L_5N_1^4}{\in^2}\Big\}+b^2\Big\{L_1L_2L_4(\bar{j}_1L_4+L_1L_3)$$

$$+L_1^2L_2L_4(L_3+\tau_1L_4)-\frac{L_1L_2L_4L_5N_1^2}{\in}+L_1L_2L_4^2\bar{j}_1+\frac{L_1L_4N_1^2M_2P_r}{\in}-\frac{L_2N_1^3\tau_1}{\in^2}x_1R_1N_3'(1+M_1)$$

$$-\frac{2L_2N_1^3\tau_1}{\in^2}x_1R_1N_3'(1+M_1)-\frac{L_1L_2L_4L_5N_1^2}{\in}+\frac{N_1^3\tau_1}{\in^2}x_1R_1M_1N_3'+\frac{2N_1^3\tau_1}{\in^2}x_1R_1M_1N_3'+\frac{L_1L_4N_1^2M_2P_r}{\in}\Big\}$$

$$+b\left\{\frac{L_1L_2\tau_1N_1^2}{\in}\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)\right)+\frac{L_1L_2N_1^2}{\in}\begin{pmatrix}2\tau_1x_1R_1(1+M)(1-M_2)\\+\tau_1x_1R_1h(1+M_1)\end{pmatrix}\right.$$

$$+L_1^2L_2L_4^2L_5+\frac{L_2\bar{j}_1N_1^2}{\in}\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)\right)-\frac{L_1\tau_1N_1^2}{\in}\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)$$

$$-\frac{2L_1N_1^2}{\in}\tau_1x_1R_1M_1(1-M_2)-\frac{L_1N_1^2}{\in}\tau_1x_1R_1M_1h-L_1^2L_4^2M_2P_r-\frac{\bar{j}_1N_1^2}{\in}\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)$$

$$+\frac{L_2N_1}{\in}x_1R_1N_3'(1+M_1)\left(\bar{j}_1L_4+L_1L_3\right)+\frac{2L_1L_2L_4N_1\tau_1}{\in}x_1R_1N_3'(1+M_1)-\frac{N_1x_1R_1M_1N_3'}{\in}(\bar{j}_1L_4+L_1L_3)$$

$$\left.-\frac{2L_1L_4N_1\tau_1}{\in}x_1R_1M_1N_3'+\frac{L_1^2L_2\tau_1T_{A_1}}{\in^2}+\frac{L_2T_{A_1}}{\in^2}\left(2\bar{j}_1L_1^2+2\bar{j}_1L_1\right)\right\}+\left\{\begin{array}{l}-L_1L_2\left(\bar{j}_1L_4+L_1L_3\right)\{x_1R_1(1+M_1)(1-M_2)\\+x_1R_1h(1+M_1)\}\end{array}\right.$$

$$-L_1^2L_2L_4\left(2\tau_1x_1R_1(1+M_1)(1-M_2)+\tau_1x_1R_1h(1+M_1)\right)-L_1L_2L_4\bar{j}_1\left(x_1R_1(1+M_1)(1-M_2)+x_1R_1h(1+M_1)\right)$$

$$+L_1(\bar{j}_1L_4+L_1L_3)\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+L_1^2L_4\left(2\tau_1x_1R_1M_1(1-M_2)+\tau_1x_1R_1M_1h\right)$$

$$\left.+\bar{j}_1L_1L_4\left(x_1R_1M_1h+x_1R_1M_1(1-M_2)\right)+\frac{L_1^2T_{A_1}}{\in^2}\left(L_2L_5-M_2P_r\right)\right\}$$

It is clear from equation (70) that σ_1 may be either zero or non-zero, which implies that the modes may be either non-oscillatory or oscillatory.

In the absence of dust particles $(\tau_1=0,\ h=0,\ f=0)$ and rotation $(T_{A_1}=0)$, the equation (70) becomes

$$\sigma_1\left[bL_3^2\bar{j}_1^2\left(M_2P_r-L_2L_5\right)\sigma_1^4+\sigma_1^2\left\{\frac{2L_3\bar{j}_1N_1^2b^2}{\in}\left(M_2P_r-L_2L_5\right)-4L_1L_3L_4\bar{j}_1b\left(M_2P_r-L_2L_5\right)\right.\right.$$

$$-L_4^2\bar{j}_1^2b\left(M_2P_r-L_2L_5\right)-L_1^2L_3^2b\left(M_2P_r-L_2L_5\right)+2L_1L_2L_3^2\bar{j}_1b^2$$

$$\left.+2L_2L_3L_4\bar{j}_1^2b^2-L_3\bar{j}_1^2x_1R_1(1-M_2)\left(L_2(1+M_1)-M_1\right)\right\}$$

$$+\left\{\frac{L_1L_2L_3N_1^2b^3}{\in}+\frac{2L_2L_4\bar{j}_1N_1^2b^3}{\in}+\frac{N_1^4b^3}{\in^2}\left(M_2P_r-L_2L_5\right)-2L_1L_2L_4^2\bar{j}_1b-2L_1^2L_2L_3L_4b^2\right.$$

$$-2L_1L_4N_1^2b\left(M_2P_r-L_2L_5\right)-L_1^2L_4^2b\left(M_2P_r-L_2L_5\right)-\frac{\bar{j}_1N_1^2}{\in}x_1R_1(1-M_2)b\left(L_2(1+M_1)-M_1\right)$$

$$-L_4\frac{\bar{j}_1N_1}{\in}x_1R_1N_3'b\left(L_2(1+M_1)-M_1\right)-\frac{L_1L_3}{\in}N_1x_1R_1N_3'b\left(L_2(1+M_1)-M_1\right)$$

$$\left.+2L_1L_4\bar{j}_1x_1R_1(1-M_2)\left(L_2(1+M_1)-M_1\right)+L_1^2L_2L_3x_1R_1(1+M_1)(1-M_2)\right\}=0 \quad ...(71)$$

In the absence of micro-viscous effect $(N_1=0)$ and microinertia $(\bar{j}_1=0)$, equation (71) reduces to

$$\sigma_1\left[\sigma_1^2L_1^2L_3^2b\left(L_2L_5-M_2P_r\right)+L_1^2L_4^2b\left(L_2L_5-M_2P_r\right)-2L_1^2L_2L_3L_4b^2+L_1^2L_2L_3x_1R_1(1+M_1)(1-M_2)\right]=0 \quad ...(72)$$

Now equation (72) yields that

$$\sigma_1=0\ \text{ when }\ \frac{\pi^2}{M_3}\left(1-\frac{M_2P_r}{P_r'}\right)<a^2<\pi^2\left(1-\frac{\in P_r}{2K_1'}\right)$$

Thus, in the absence of dust particles, rotation, micro-viscous effect and microinertia, the sufficient condition for non-oscillatory modes is given by

$$\frac{\pi^2}{M_3}\left(1-\frac{M_2P_r}{P_r'}\right)<a^2<\pi^2\left(1-\frac{\in P_r}{2K_1'}\right) \quad ...(73)$$

X. OBSERVATIONS

In stationary convection, the variations of thermal Rayleigh number *R*, with respect to the variations of Medium permeability (K_1'), (see fig. 2 to 3); Non-Buoyancy magnetization (M_3), (see fig. 4 to 5): Micropolar heat conduction parameter ($\bar{\delta}$) (see fig. 6 to 7); Dust particles parameter (h_1), (see fig. 8); and Taylor number (T_{A_1}) , (see fig. 9); respectively have been predicted by the graphs given below:

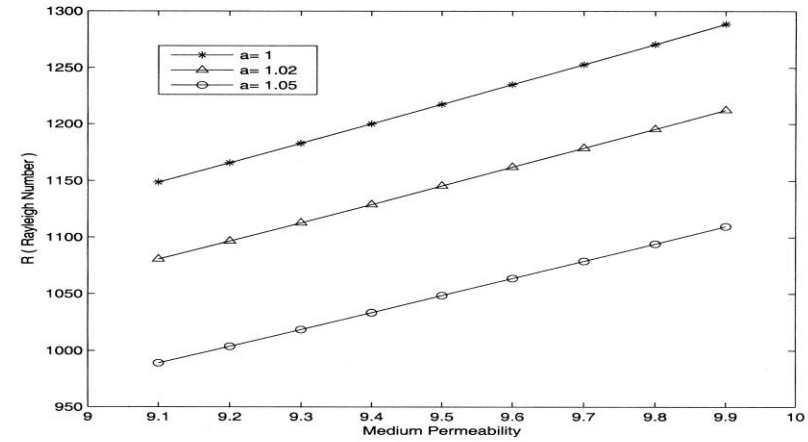

Fig. 2: Marginal instability curve for the variation of R vs κ_1' for $\in$=0.5, T_{A_1} =100, h_1=1.2, N_1=0.2, N'_2=2, N'_3=0.5, M_1=100, M_3=5.

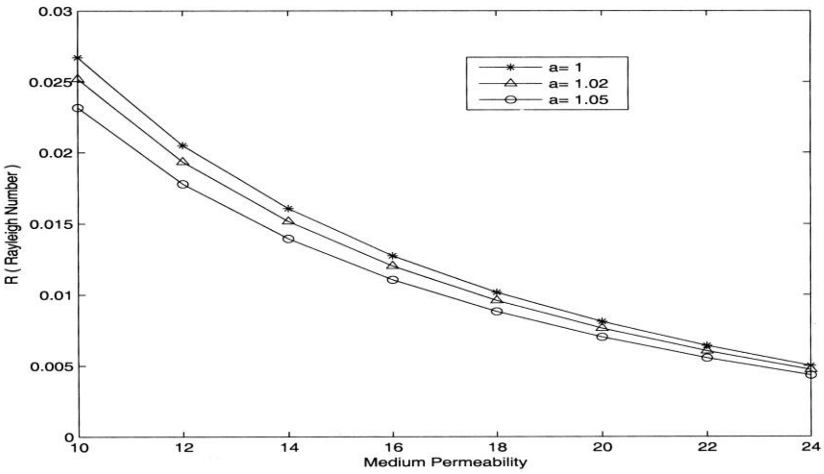

Fig. 3: Marginal instability curve for the variation of R vs κ_1' for $\in$=0.5, T_{A_1} =0, h_1=1.2, N_1=0.2, N'_2=2, N'_3=0.5, M_1=100, M_3=5.

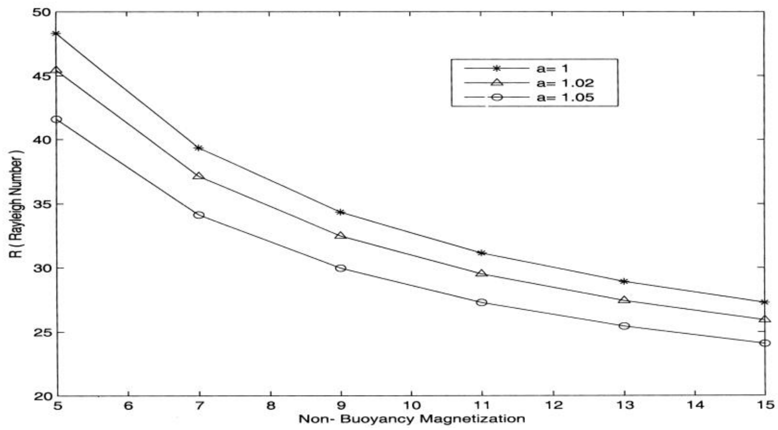

Fig. 4: Marginal instability curve for the variation of R vs M_3 for $\in$=0.5, T_{A_1} =100, h_1=1.2, N_1=0.2, N'_2=2, N'_3=0.3, M_1=100, K'_1=0.5.

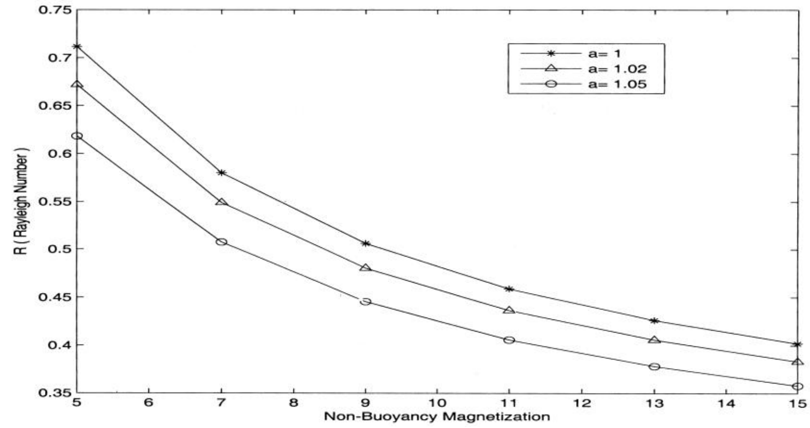

Fig. 5: Marginal instability curve for the variation of R vs M_3 for $\in$=0.5, h_1=1.2, N_1=0.2, N'_2=2, N'_3=0.3, M_1=100, K'_1=0.5, T_{A_1} =0.

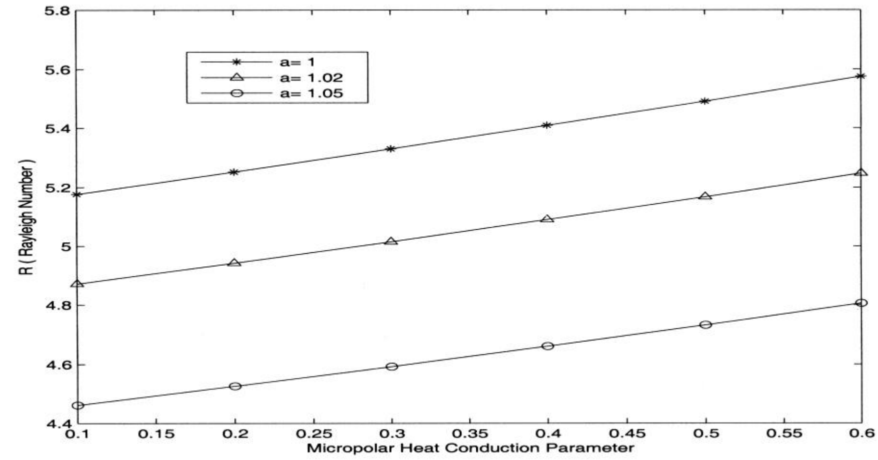

Fig. 6: Marginal instability curve for the variation of R vs N'_3 for $\in$=0.5, h_1=1.2, T_{A_1} =10, N_1=0.2, N'_2=2, M_1=100, K'_1=0.5, M_3=5.

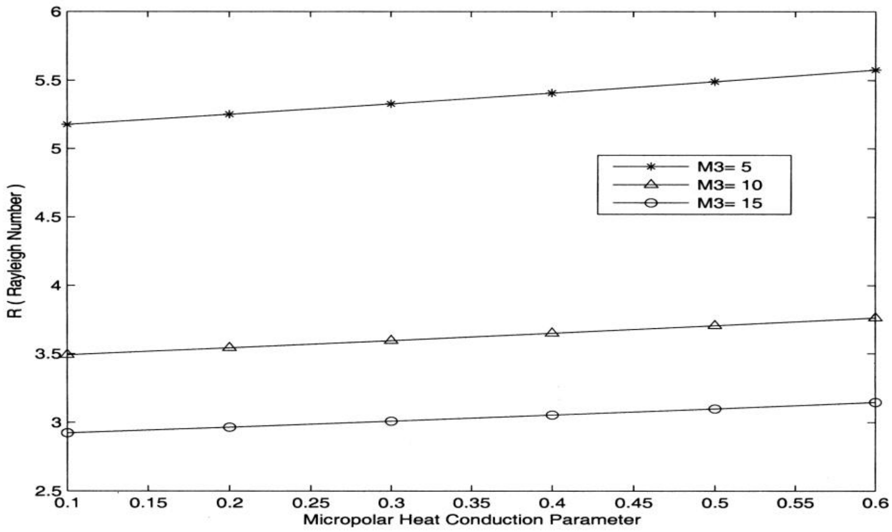

Fig. 7: Marginal instability curve for the variation of R vs N'_3 for $\in$=0.5, h_1=1.2, T_{A_1} =10, N_1=0.2, N'_2=2, M_1=100, a=1, K'_1=0.5.

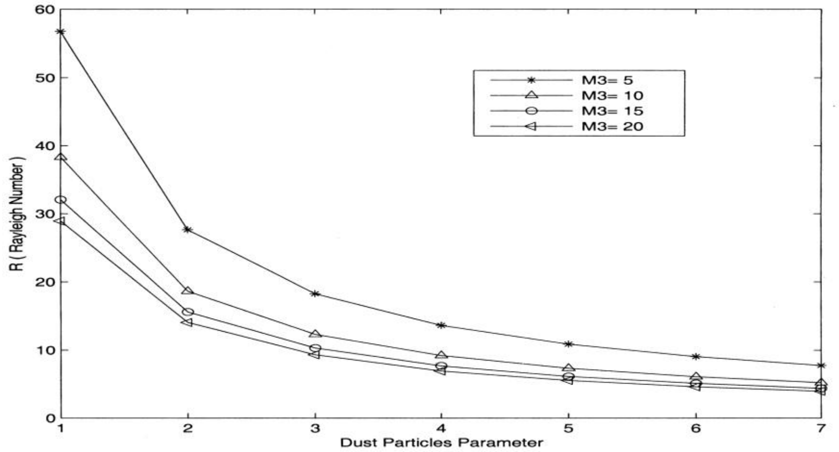

Fig. 8: Marginal instability curve for the variation of R vs h_1 for $\in$=0.5, T_{A_1} =100, N_1=1.2, N'_2=2, N'_3=0.3, K'_1=0.5, M_1=100, a=1.

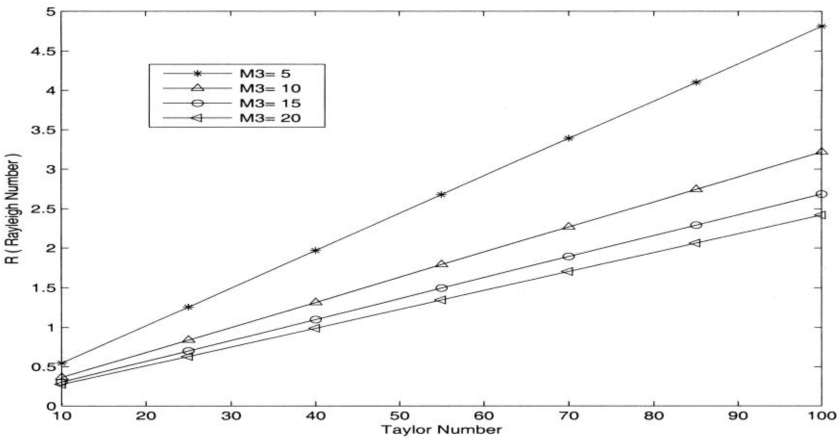

Fig. 9: Marginal instability curve for the variation of R vs T_{A_1} for $\in$=0.5, N_1=0.2, N'_2=2, N'_3=0.3, M_1=1000, K'_1=0.5, h_1=1.2, a=1.

XI. CONCLUSIONS

For Stationary Convection:

1. The medium permeability has stabilizing effect when

$$T_{A_1} > \frac{N_1 \in^2}{N_2'}\left(\frac{1+N_1}{K_1'}\right)^2, \quad \max.\left\{\frac{1}{2}\left(1+\frac{1}{N_1}\right)^2, \frac{1}{N_1}\left(1+\frac{1}{N_1}\right)\right\} < \frac{K_1'}{\in} < \frac{2N_2'}{N_1} \quad \text{and} \quad h_1 N_2' > \frac{N_1 N_3'}{\in}$$ **(see fig. 2)**

2. In the absence of rotation, the medium permeability has destabilizing effect when

$$\frac{N_2'}{2N_1} < \frac{K_1'}{\in} < \frac{2N_2'}{N_1^2} \quad \text{and} \quad h_1 N_2' > \frac{N_1 N_3'}{\in}$$ **(see fig. 3)**

3. The non-buoyancy magnetization has destabilizing effect when

$$K_1' < \frac{2\in}{b} \quad \text{and} \quad h_1 > \frac{N_1 N_3'}{\in N_2'}$$ **(see fig. 4)**

Also the destabilizing behaviour of non-buoyancy magnetization remains unaffected by rotation under the same conditions **(see fig. 5)**

4. The coupling parameter has a stabilizing effect when

$$T_{A_1} < \frac{b \in^2}{K_1'^2},\ K_1' < \frac{2 \in}{b},\ \frac{N_1 N_3'}{N_2' \in} < h_1 < \frac{b N_3'}{2 \in}$$

5. The spin-diffusion parameter has a destabilizing effect when

$$\frac{N_1 N_3'}{N_2' \in} < h_1 < \frac{N_3'}{K_1'} \text{ and } K_1' < \frac{2 \in}{b}$$

6. The micropolar heat conduction parameter has a stabilizing effect when $K_1' < \frac{2 \in}{b}$ **(see fig. 6 & 7)**
7. The dust particles has a destabilizing effect when $K_1' < \frac{2 \in}{b}$ **(see fig. 8)**
8. The rotation has a stabilizing effect when $K_1' < \frac{2 \in}{b}$ and $h_1 > \frac{N_1 N_3'}{\in N_2'}$ **(see fig. 9)**
9. In case of stationary convection, the micropolar ferromagnetic fluid behaves like an ordinary micropolar fluid for sufficiently large values of M_1 and M_3.
10. The magnetic thermal Rayleigh number R_m has its minimum value when equation (70) is identically satisfied.

For Oscillatory Convection:
In the absence of dust particles, rotation, micro-viscous effect and microinertia, the sufficient condition for non-oscillatory modes is given by

$$\frac{\pi^2}{M_3}\left(1 - \frac{M_2 P_r}{P_r'}\right) < a^2 < \pi^2\left(1 - \frac{\in P_r}{2 K_1'}\right)$$

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